Recent Results on the Longest Common Substring Problem

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Longest Common Substring

Definition (Longest Common Substring Problem)

Input: Two strings S and T of length at most n

Output: The longest string U that is a substring of both S and T

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This is not the Longest Common **Subsequence** problem

Longest Common Factor

Definition (Longest Common Factor (LCF) Problem)

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Motivation for LCF:

- An elementary string similarity measure
- Computationally easier than LCS [1,2]

[1] A. Abboud, A. Backurs, V. Vassilevska Williams:
Tight Hardness Results for LCF and Other Sequence Similarity Measures. FOCS 2015

[2] K. Bringmann, M. Künnemann: Quadratic Conditional Lower Bounds
for String Problems and Dynamic Time Warping. FOCS 2015

Early Results on LCF

History of the LCF problem:

- Donald E. Knuth conjectured no $o(n \log n)$-time solution for LCF
- $O(n)$ time LCF for $O(1)$ alphabet using the suffix tree [1]

[1] P. Weiner: Linear Pattern Matching Algorithms, SWAT 1973

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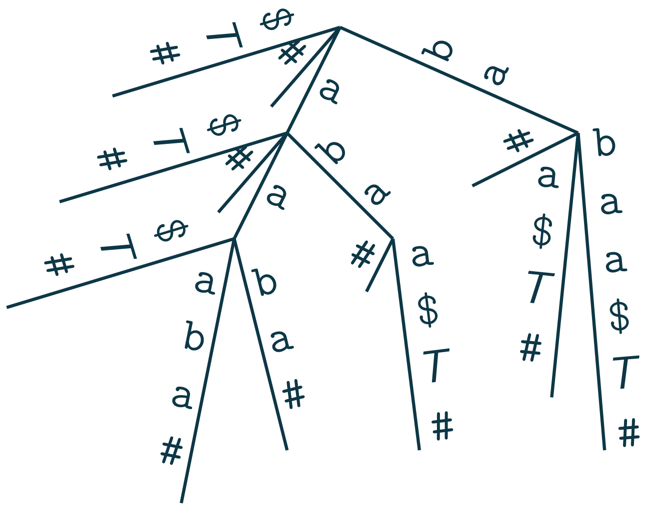

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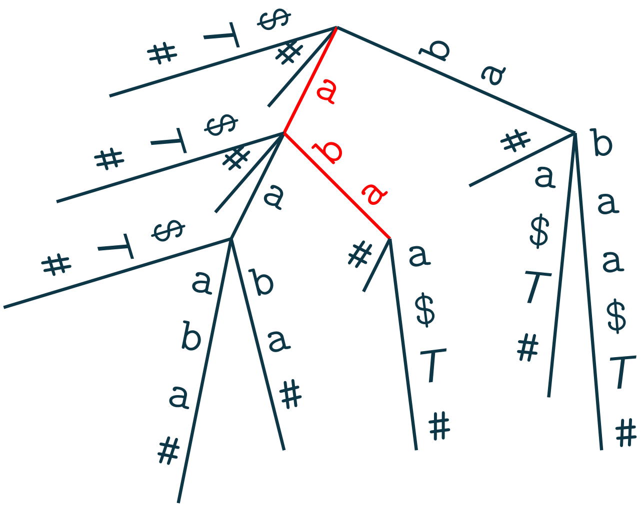

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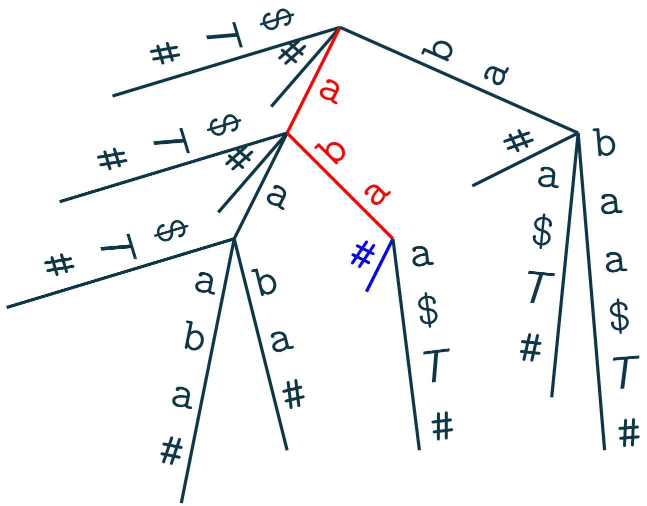

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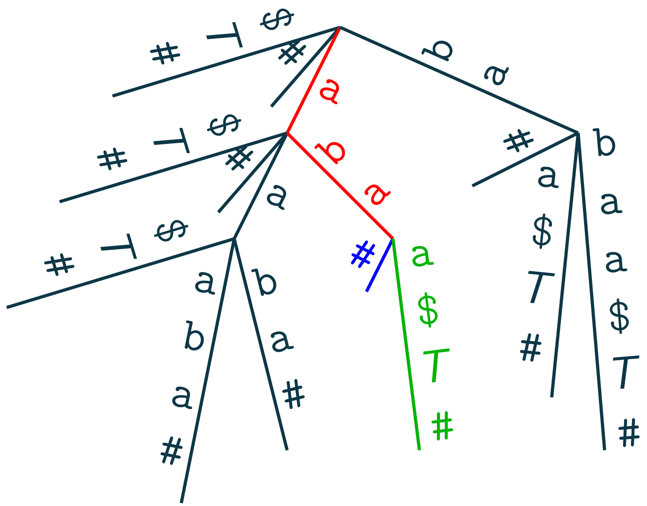

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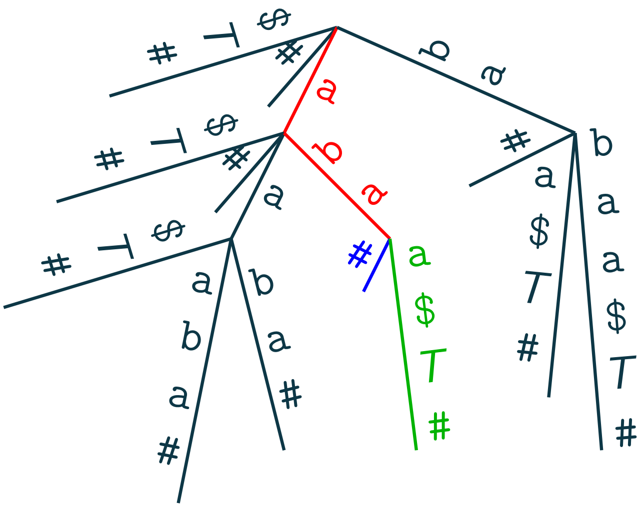

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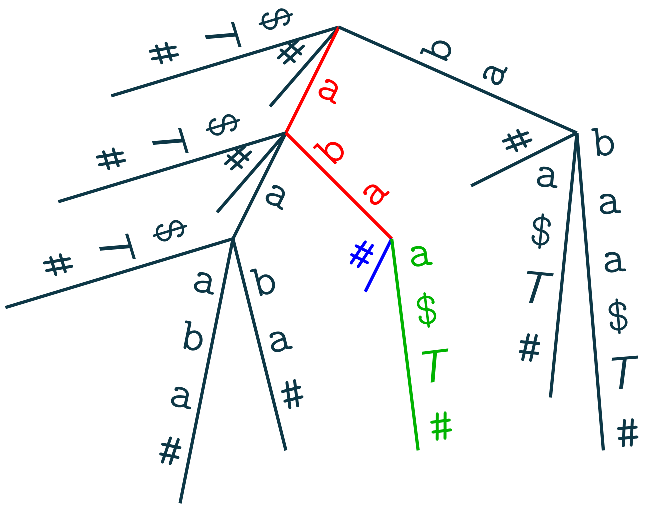

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- $O(n)$ time for integer alphabet $[0, n^{O(1)}]$, see [2]
- $O(n)$ time for many strings of total length $O(n)$ [3]

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[2] M. Farach: Optimal Suffix Tree Construction with Large Alphabets, FOCS 1997

[3] L.C.K. Hui. Color set size problem with applications to string matching. CPM 1992

Even Faster LCF

Packed setting:

- $O(1)$-sized alphabet, say `A`, `C`, `G`, `T`
- A string can be represented with $O(n/\log n)$ machine words

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Theorem ([1])

LCF on packed strings can be solved in $O(n/\sqrt{\log n})$ time.

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Relation to packed indexing:

- $O(n\log\sigma/\log n)$-sized index can be constructed in $O(n\log\sigma/\sqrt{\log n})$ time [2,3]
- No direct computation of packed LCF from packed index is known.

[1] P. Charalampopoulos, T. Kociumaka, S.P. Pissis, **R**: Faster Algorithms for LCF. ESA 2021

[2] D. Kempa, T. Kociumaka: String Synchronizing Sets: Sublinear-time BWT Construction and Optimal LCE Data Structure. STOC 2019

[3] J.I. Munro, G. Navarro, Y. Nekrich: Text Indexing and Searching in Sublinear Time. CPM 2020

Some Details on Packed LCF

Assumptions for the talk:

- Alphabet of size $\sigma = O(1)$
- A number in $[0, n)$ encodes $\log_\sigma n = \Theta(\log n)$ letters.
- A string of length n is represented by $\Theta(n/\log n)$ numbers in $[0, n)$.

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Three cases:

- Short LCF: $\leq \frac{1}{3}\log_\sigma n$
- Medium LCF
- Long LCF: $\geq \log^4 n$ [1]

We aim at $O(n/\sqrt{\log n})$ time.

[1] P. Charalampopoulos, M. Crochemore, C.S. Iliopoulos, T. Kociumaka, S.P. Pissis, **R**, W. Rytter, T. Waleń: Linear-Time Algorithm for Long LCF with k Mismatches. CPM 2018

Plan of Presentation

- Packed LCF:
  - Short LCF
  - Long LCF
  - Medium-length LCF
- Approximate LCF
- Small-space LCF
- Compressed LCF
- Dynamic LCF
- Internal LCF

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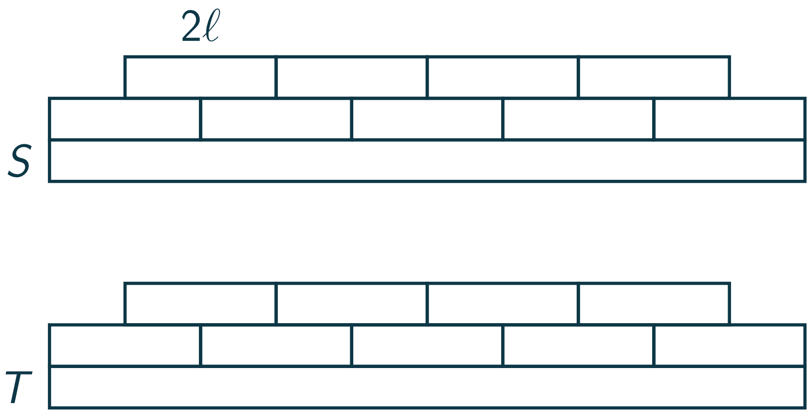
Short LCF ($\leq \frac{1}{3}\log_\sigma n$)

- Let $\ell = \frac{1}{3}\log_\sigma n$. Split each string into overlapping substrings of length 2ℓ.
- The LCF is a substring of one length-2ℓ substring in S and in T.



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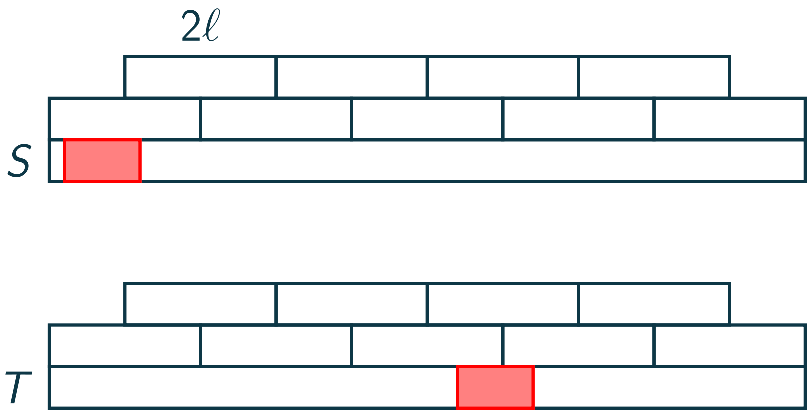

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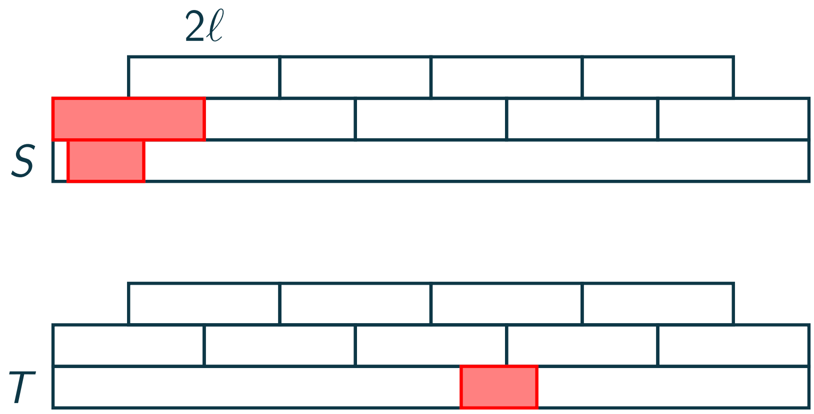

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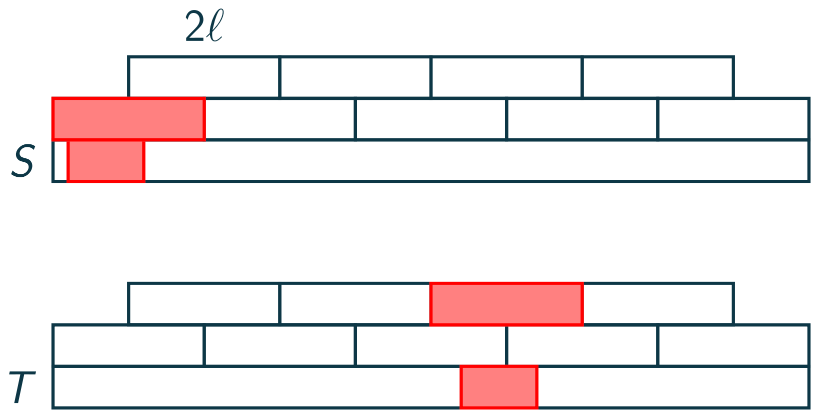

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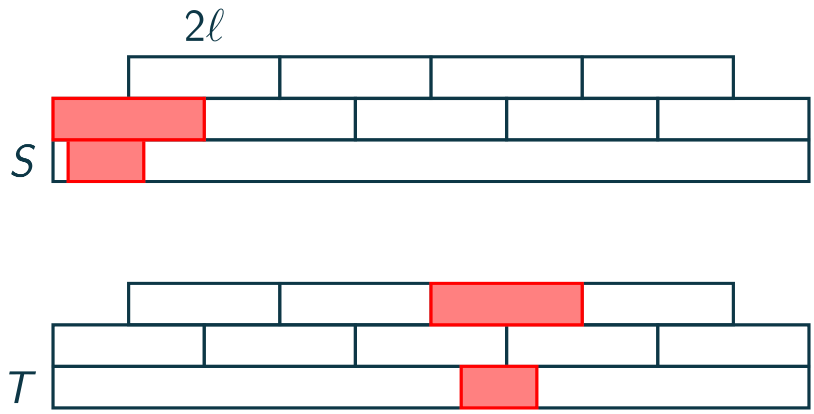

- $\leq \sigma^{2\ell} = \sigma^{\frac{2}{3} \log_\sigma n} = n^{2/3}$ distinct length-2ℓ substrings of S and T
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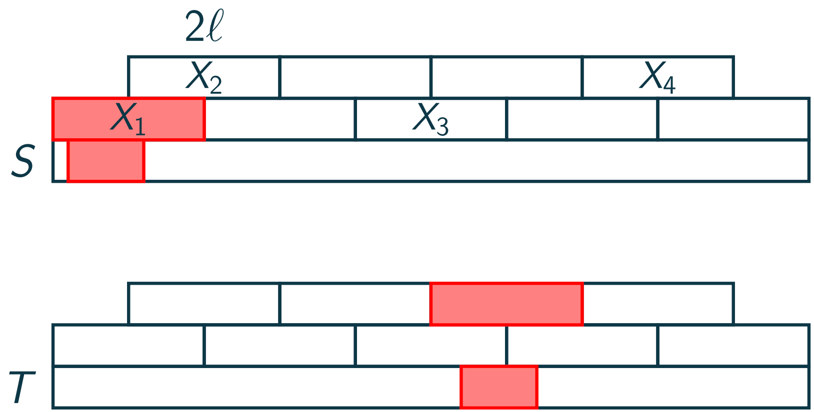

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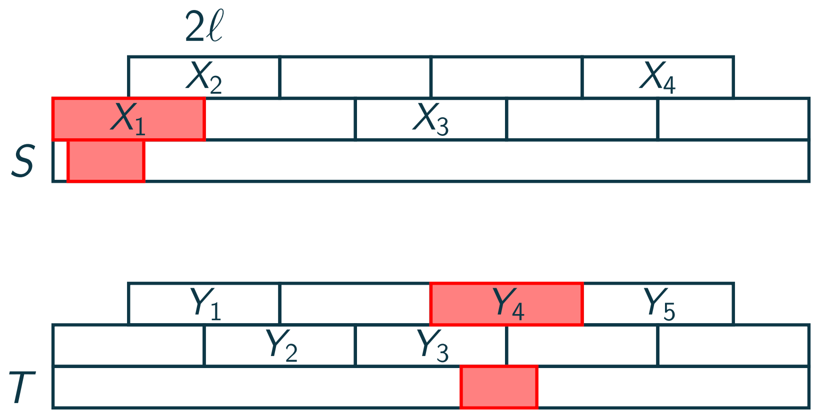

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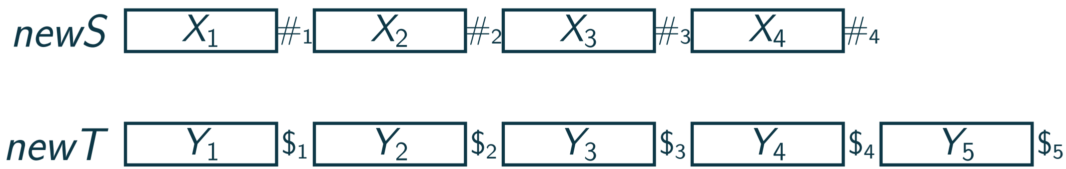

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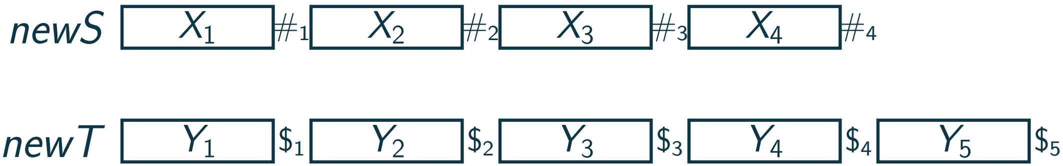

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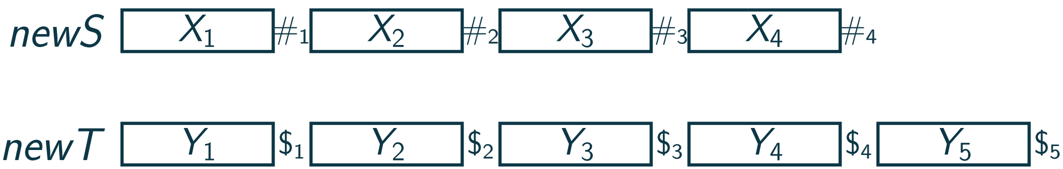

$|newS|,\ |newT| \leq n^{2/3}(\frac{2}{3}\log n + 1)$

LCF of $newS$ and $newT$ can be computed in $O(n^{2/3}\log n) = o(n/\log n)$ time.

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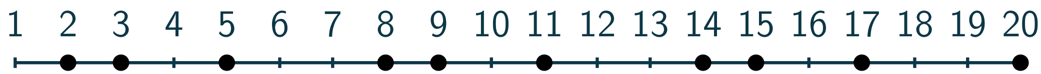
Difference Cover

Definition (Difference cover)

A set D is a d-cover if there is a function *shift* such that for any $i, j > 0$ we have $0 \leq \mathit{shift}(i,j) < d$ and $i + \mathit{shift}(i,j)$, $j + \mathit{shift}(i,j) \in D$.

Example. $\{2, 3, 5, 8, 9, 11, 14, 15, 19, 20, \ldots\}$ is a 6-cover:



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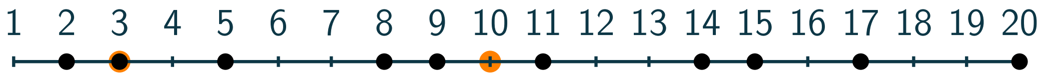

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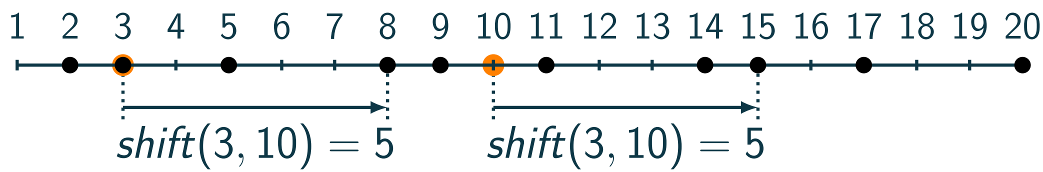

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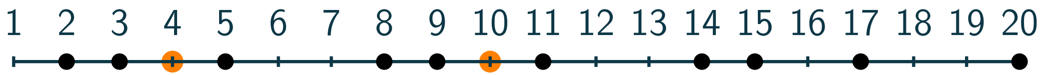

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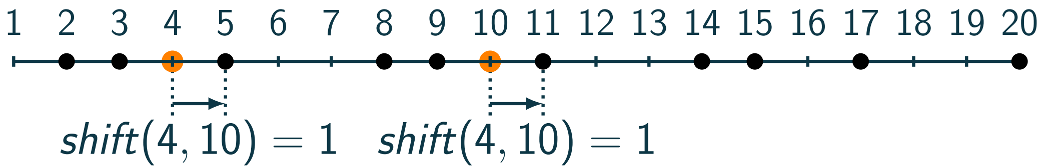

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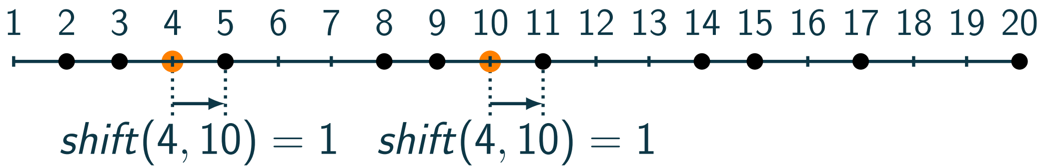

Lemma ([1])

A d-cover D such that $D \cap [1, n]$ is of size $O(n/\sqrt{d})$ can be constructed in $O(n/\sqrt{d})$ time.

[1] M. Maekawa: A Square Root N Algorithm for Mutual Exclusion in Decentralized Systems. ACM Trans. Comput. Syst., 1985

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- Use the lemma for ST, $2n$ pos., and $d = \ell$; the ℓ-cover has size $O(n/\log^2 n)$.
- Let $\ell = \log^4 n$. An ℓ-cover is useful in computing LCF of length $\geq \ell$:

```
c b b a a b c a b c a b c b a
  •     •     •     •
  2     5     8     11

d a a b a a b c b b a
            •     •
            6     9
```

subset of 4-cover

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c b b a a b c a b c a b c b a

2 5 8 11

d a a b a a b c b b a

6 9

subset of 4-cover

Long LCF ($\geq \log^4 n$)

Definition (Difference cover)

A set D is a d-cover if there is a function *shift* such that for any $i, j > 0$ we have $0 \leq shift(i,j) < d$ and $i + shift(i,j), j + shift(i,j) \in D$.

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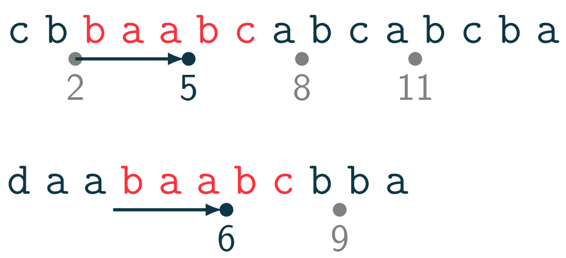

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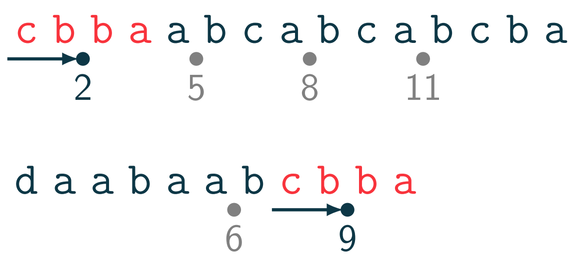

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```
c b b a a b c a b c a b c b a
  •     •     •     •
  2     5     8     11

d a a b a a b c b b a
          •     •
          6     9
```

subset of 4-cover

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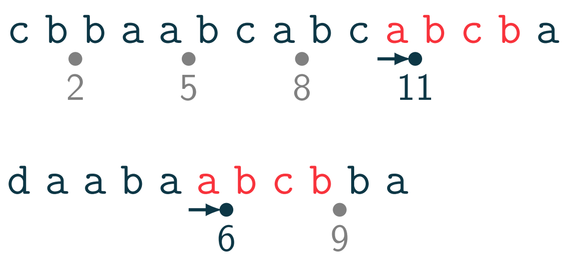

subset of 4-cover

Long LCF ($\geq \log^4 n$)

- LCF anchored at a pair of elements of a difference cover

```
1 c b b a a b c a b c a b c b a 1
     •     •     •     •
     2     5     8     11

2 d a a b a a b c b b a 2
             •     •
             6     9
```

Long LCF ($\geq \log^4 n$)

- LCF anchored at a pair of elements of a difference cover

```
1 c b b a a b c a b c a b c b a 1
       •     •     •     •
       2     5     8     11

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               •     •
               6     9
```

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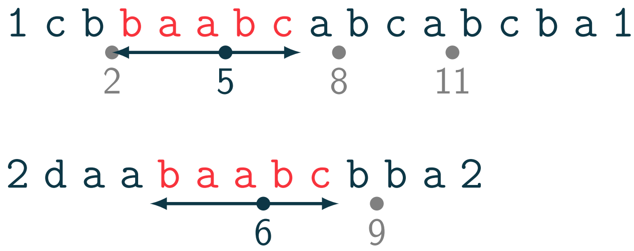

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6 9

Reversed prefixes:

2_S bc1
5_S aabbc1
8_S acbaabbc1
11_S acbacbaabbc1
6_T aabaad2
9_T bcbaabaad2

Suffixes:

2_S baabcabcabcba1
5_S bcabcabcba1
8_S bcabcba1
11_S bcba1
6_T bcbba2
9_T ba2

Long LCF ($\geq \log^4 n$)

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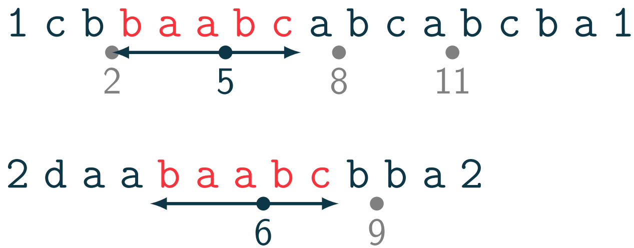

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6 9

Reversed prefixes:		Suffixes:		
2_S	bc1	2_S	baabcabcabcba1	
5_S	aabbc1	5_S	bcabcabcba1	
8_S	acbaabbc1	8_S	bcabcba1	$\mathcal{F}_S$
11_S	acbacbaabbc1	11_S	bcba1	
6_T	aabaad2	6_T	bcbba2	$\mathcal{F}_T$
9_T	bcbaabaad2	9_T	ba2	

MaxPairLCP problem:

Input: two families of pairs of strings, $\mathcal{F}_S$ and $\mathcal{F}_T$, of total size N

Output: $\max\{LCP(P_1, Q_1) + LCP(P_2, Q_2) : (P_1, P_2) \in \mathcal{F}_S, (Q_1, Q_2) \in \mathcal{F}_T\}$

Representation of MaxPairLCP

Reversed prefixes:

2_S `bc1`
5_S `aabbc1`
8_S `acbaabbc1`
11_S `acbacbaabbc1`
6_T `aabaad2`
9_T `bcbaabaad2`

Suffixes:

2_S `baabcabcabcba1`
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9_T `ba2`

Representation of MaxPairLCP

Reversed prefixes:

aabaad2 (6_T)

aabbc1 (5_S)

acbaabbc1 (8_S)

acbacbaabbc1 (11_S)

bc1 (2_S)

bcbaabaad2 (9_T)

Suffixes:

ba2 (9_T)

baabcabcabcba1 (2_S)

bcabcabcba1 (5_S)

bcabcba1 (8_S)

bcba1 (11_S)

bcbba2 (6_T)

Representation of MaxPairLCP

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aabbc1 (5_S)
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bc1 (2_S)
bcbaabaad2 (9_T)

Suffixes:

ba2 (9_T)
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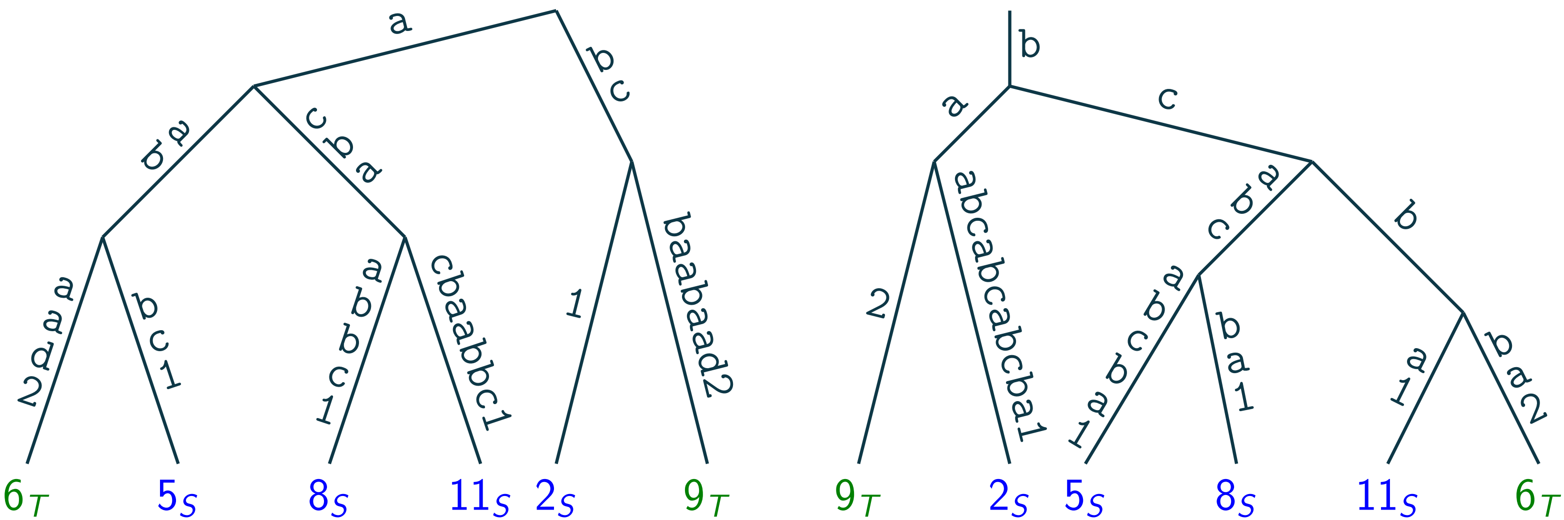
bcabcba1 (8_S)
bcba1 (11_S)
bcbba2 (6_T)



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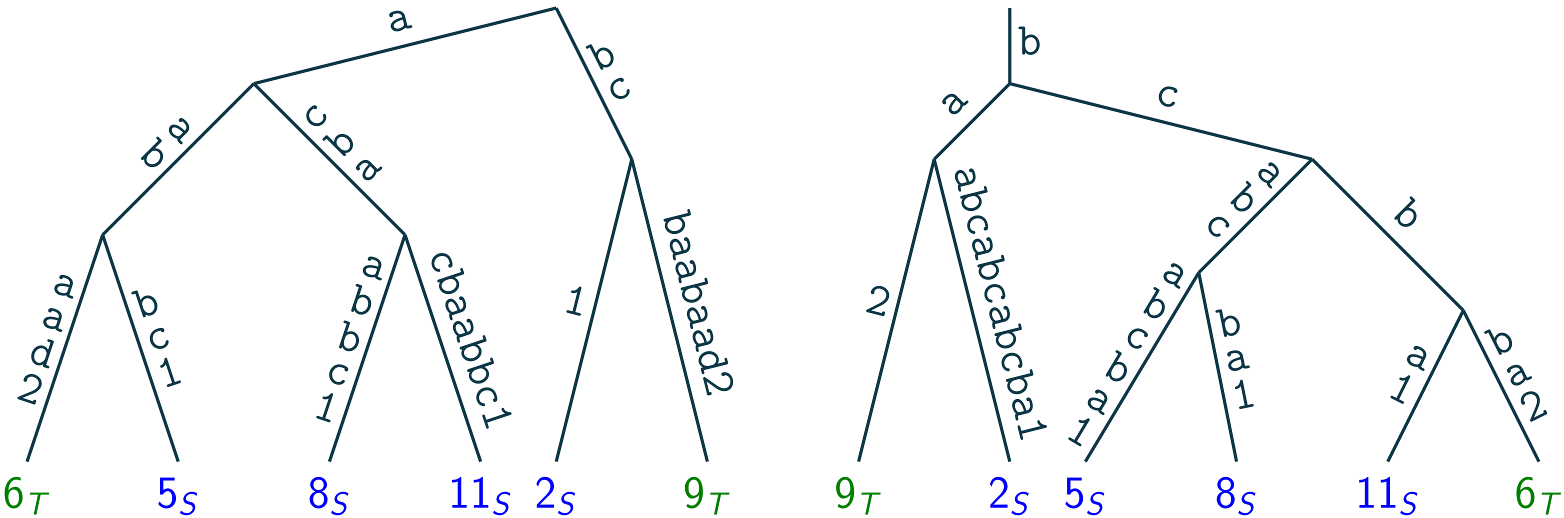

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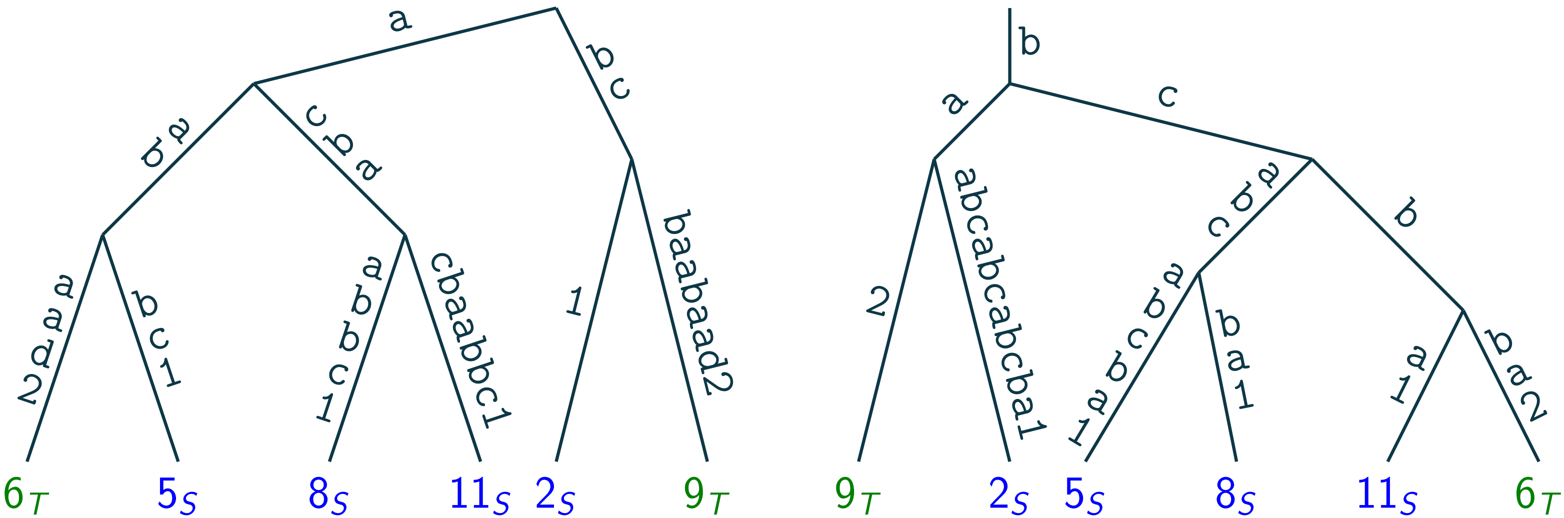

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- The tries after compactification have $O(N)$ leaves and size $O(N)$.

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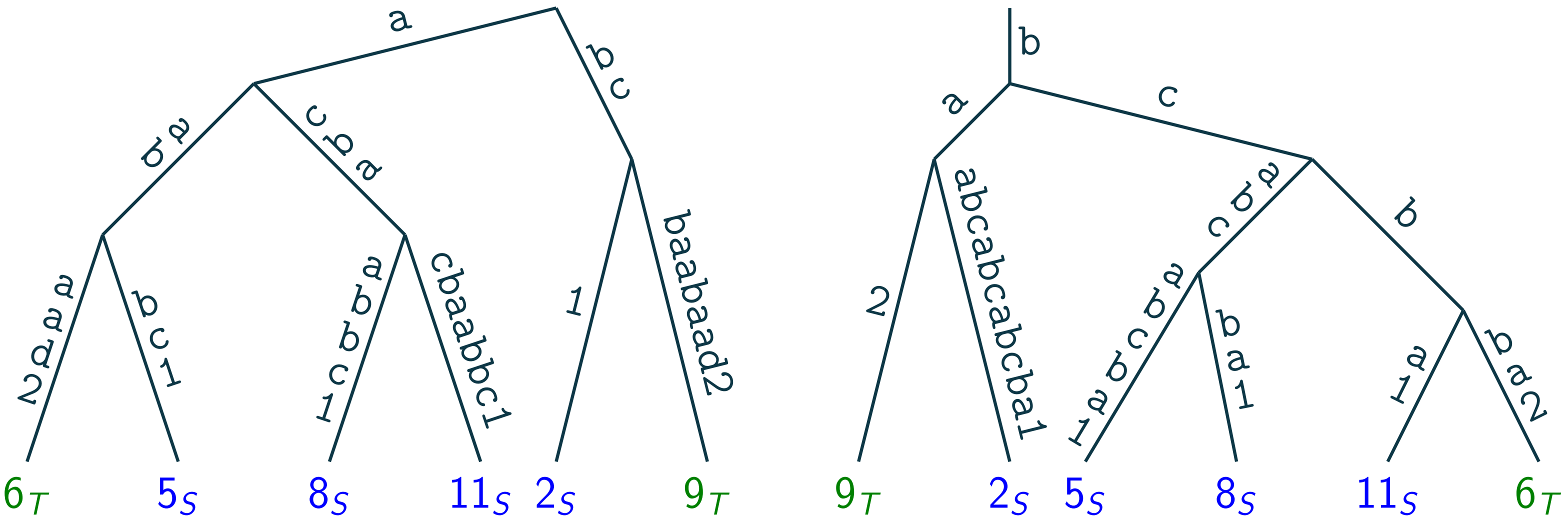

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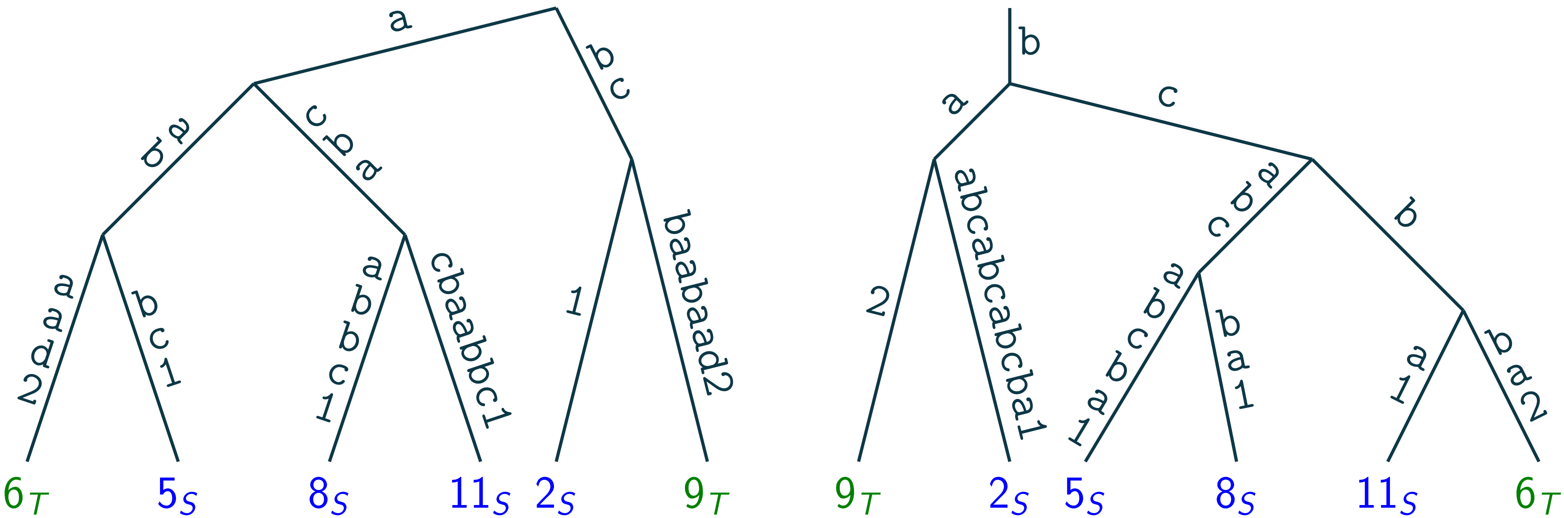
Lemma. The tries can be constructed in $O(N \log N + n/\log_\sigma n)$ time.

Representation of MaxPairLCP



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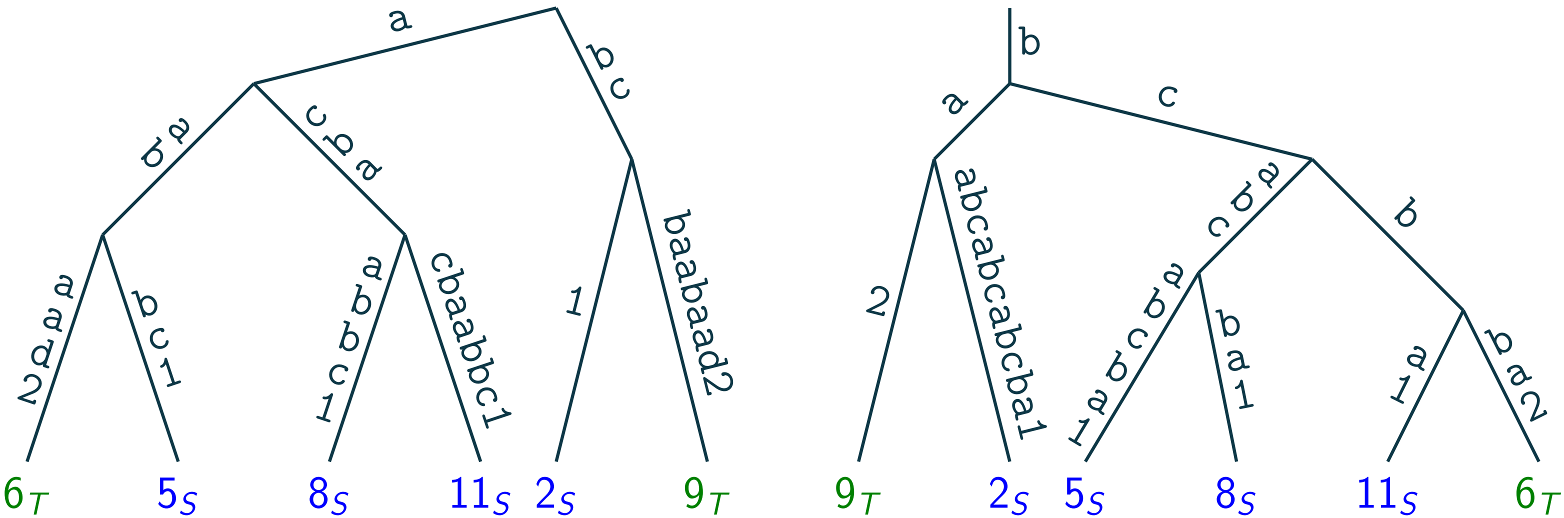
Proof. Merge Sort of strings and $O(1)$-time LCP-queries from [1].

[1] D. Kempa, T. Kociumaka: String Synchronizing Sets: Sublinear-time BWT Construction and Optimal LCE Data Structure. STOC 2019

Solution to MaxPairLCP



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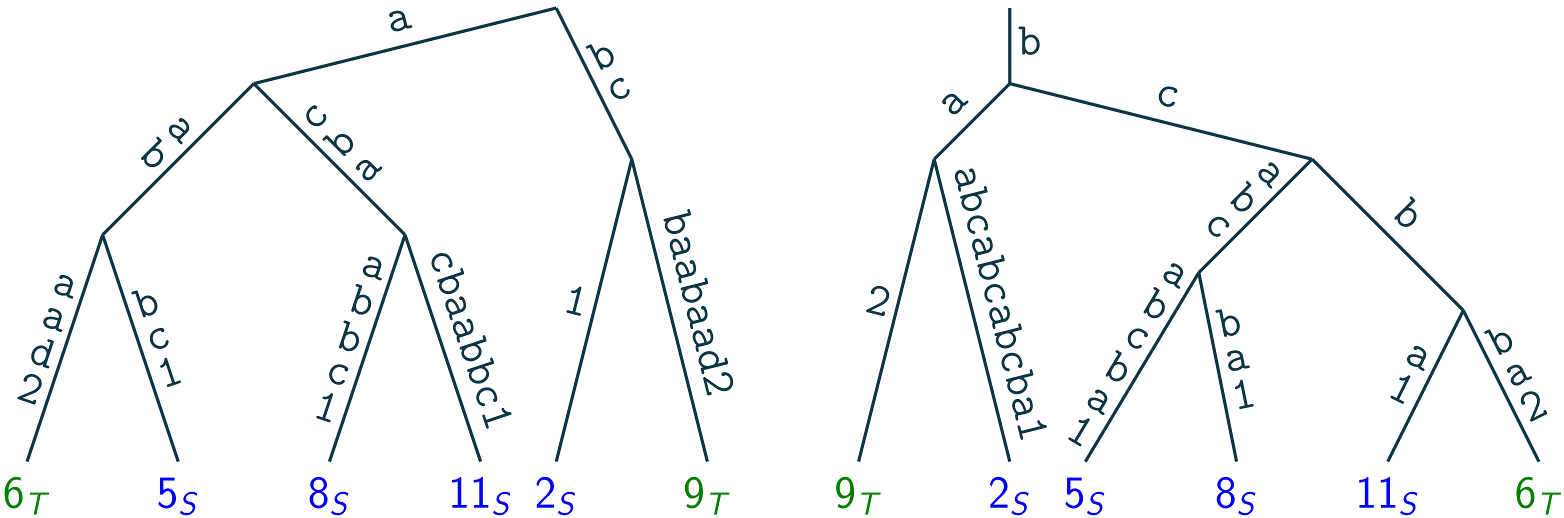

Goal:

- Nodes u, v from both tries
- Their subtrees have a pair of equal leaves, one from S and one from T
- $depth(u) + depth(v)$ is max

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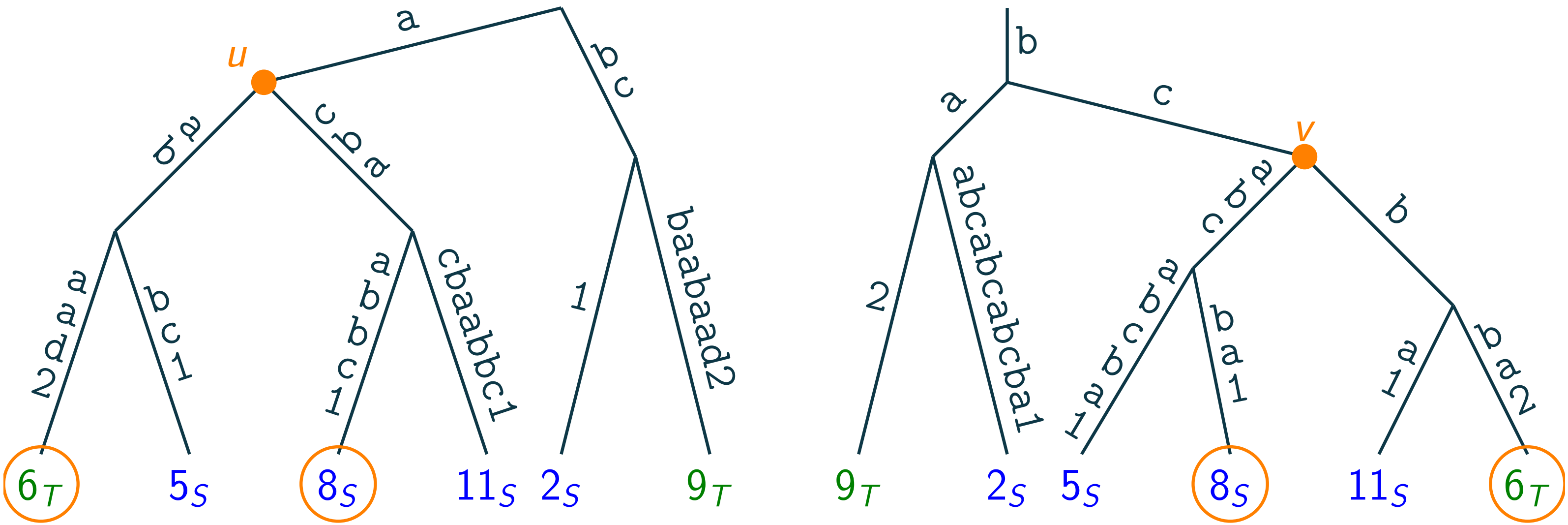

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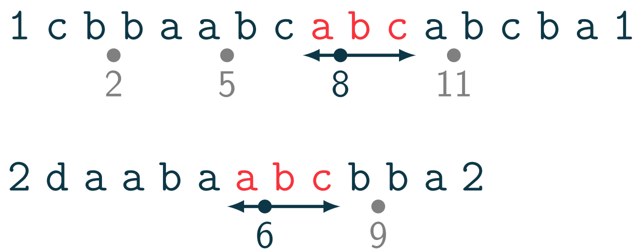

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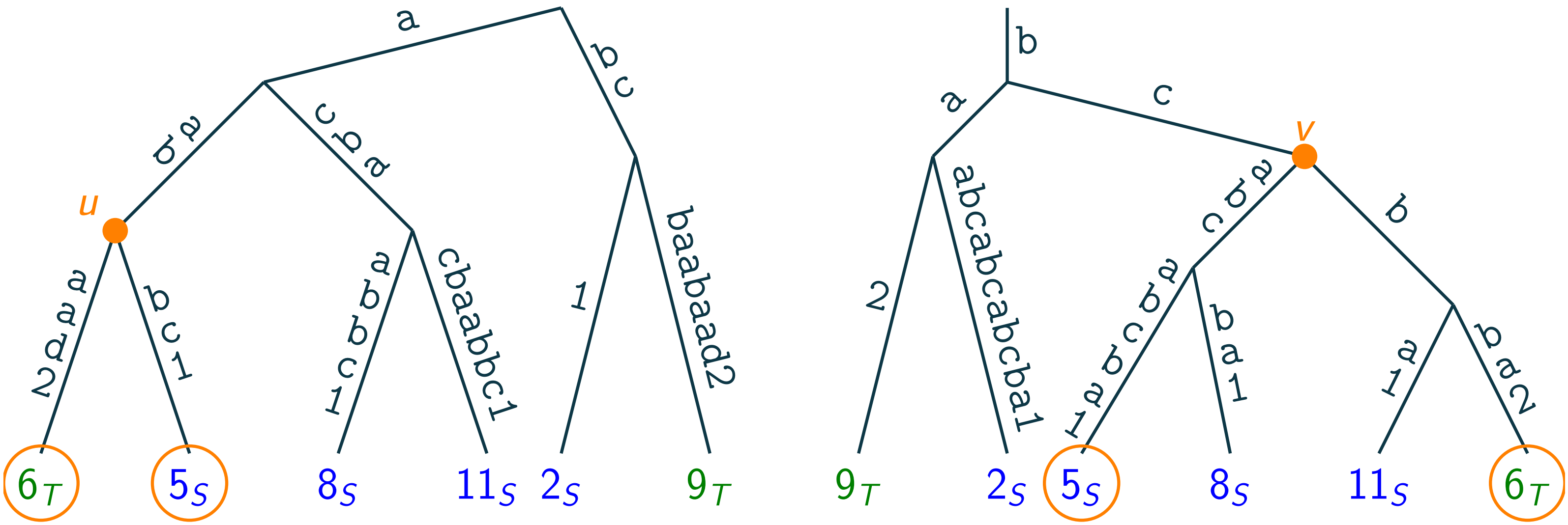

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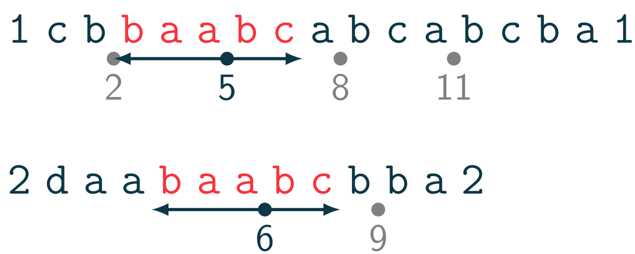

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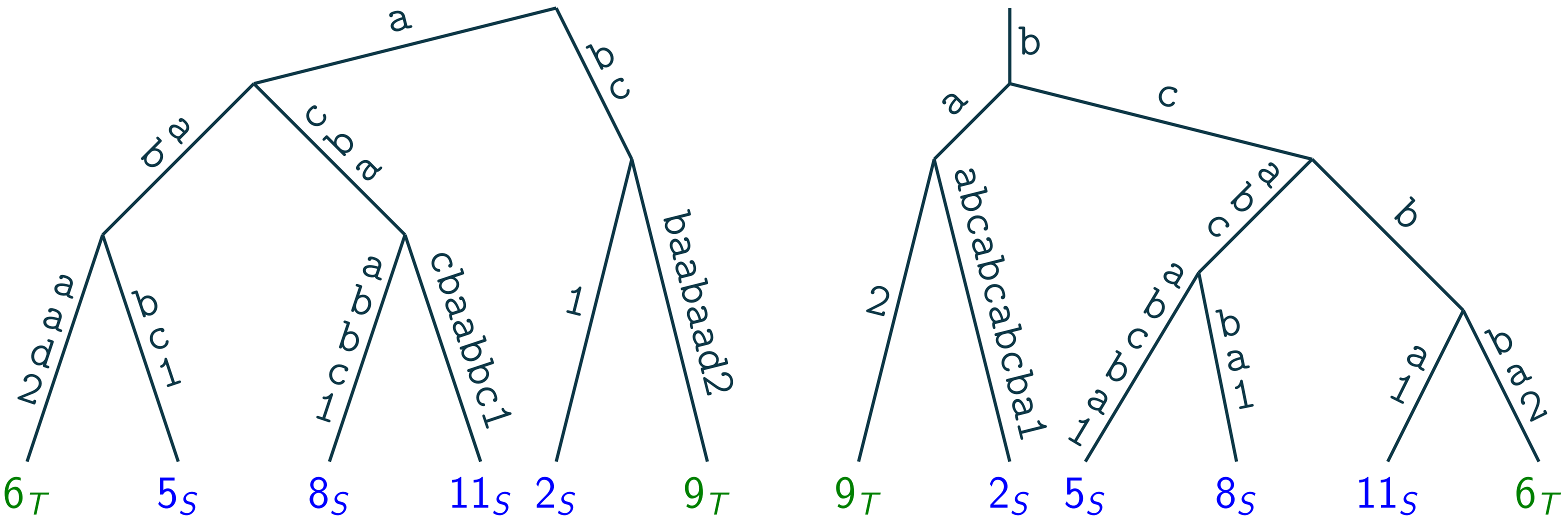

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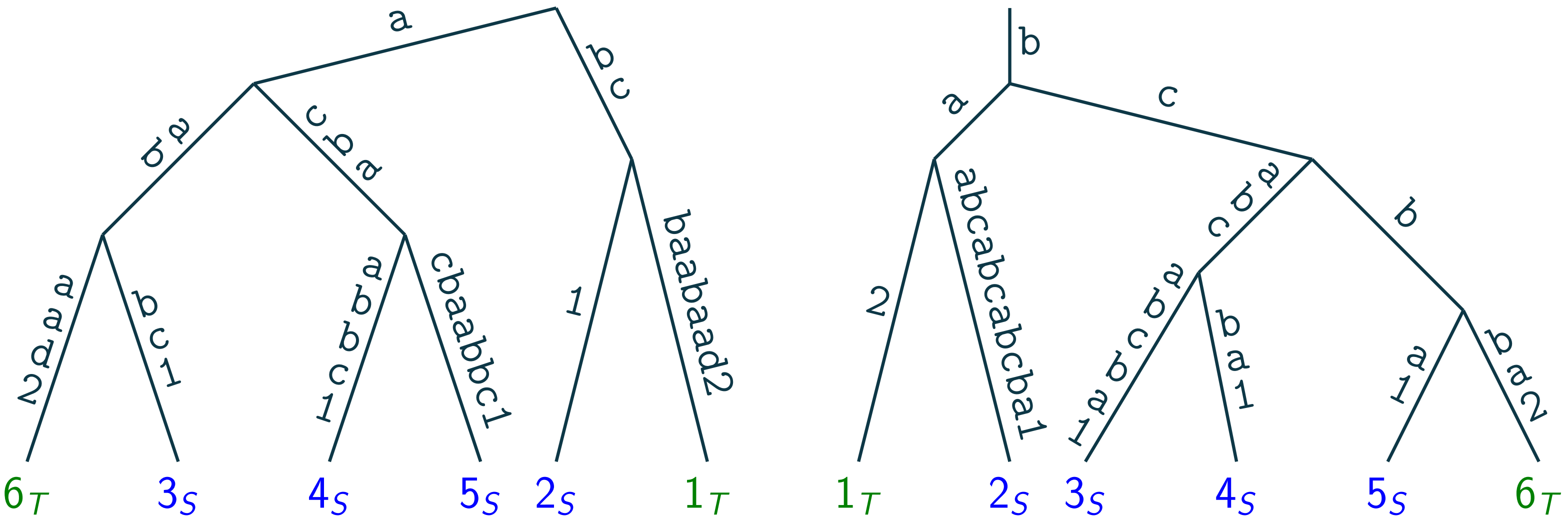

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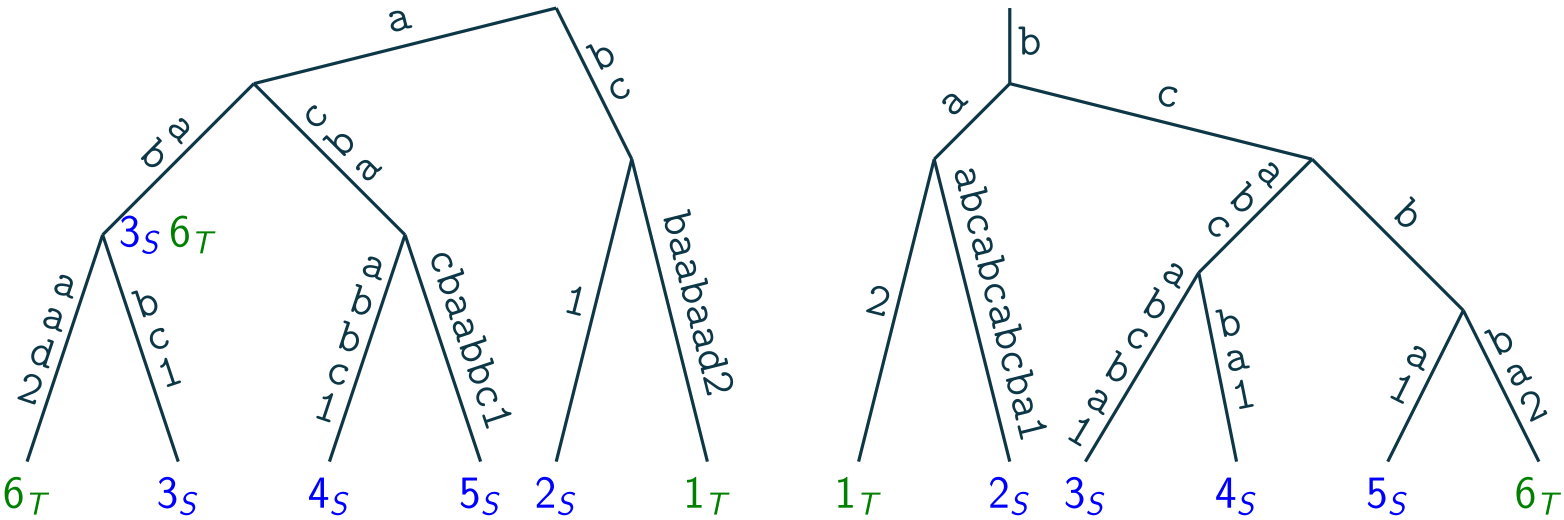

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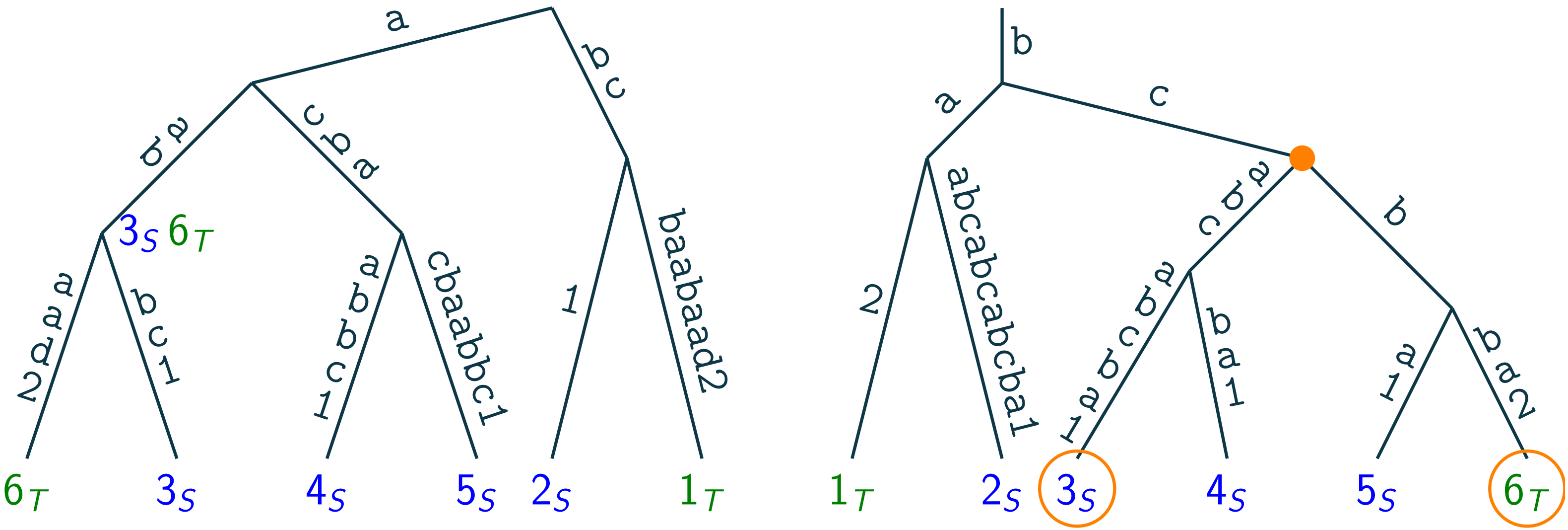

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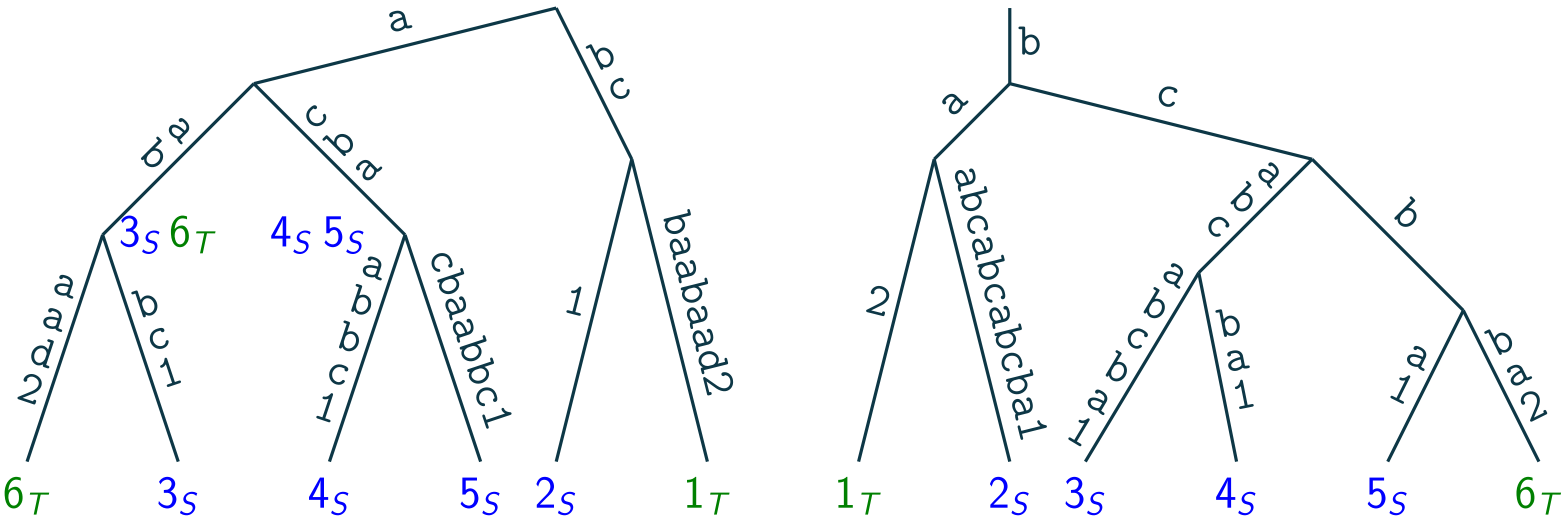

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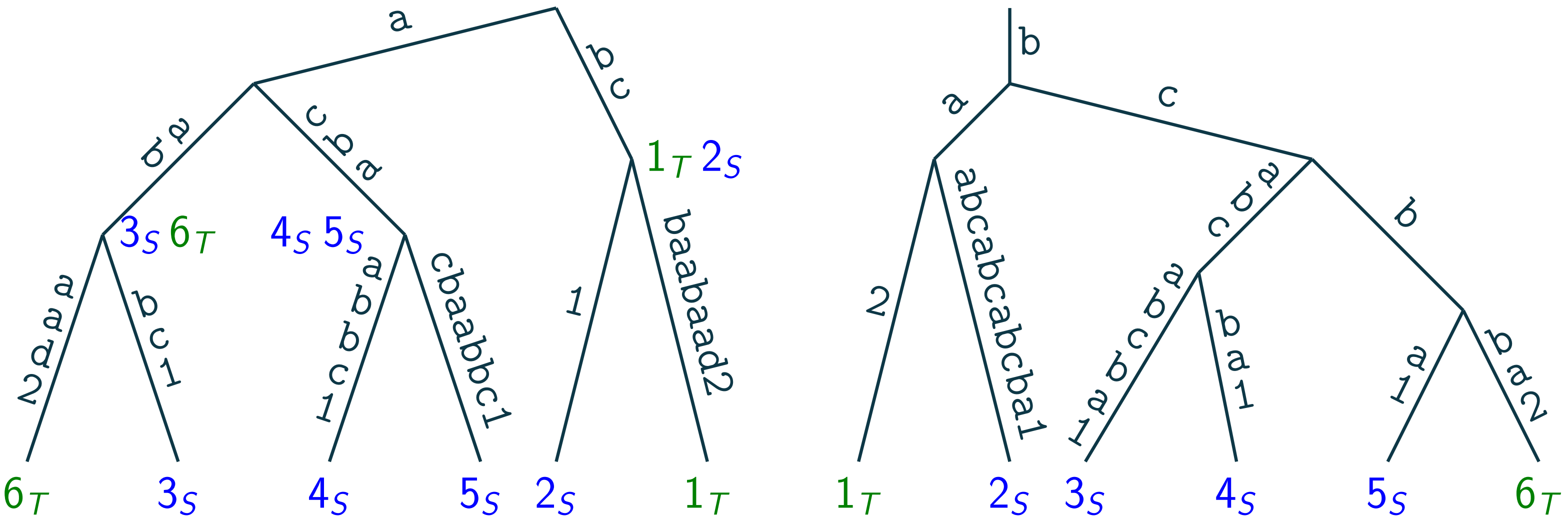

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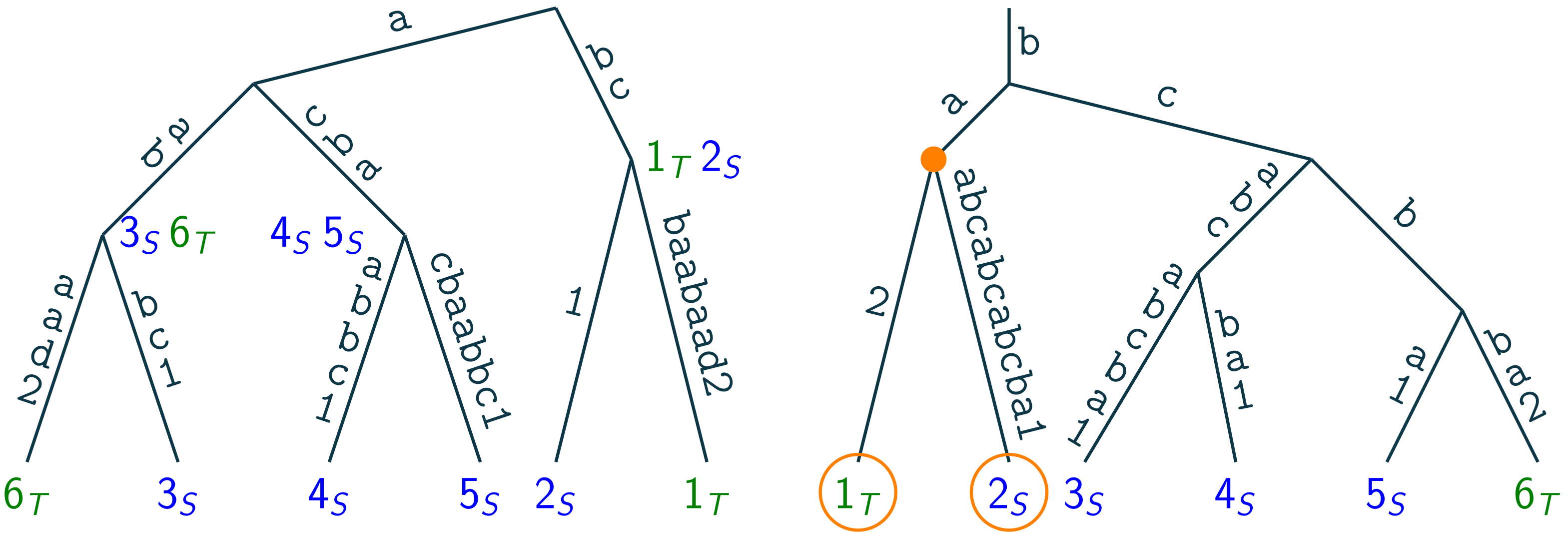

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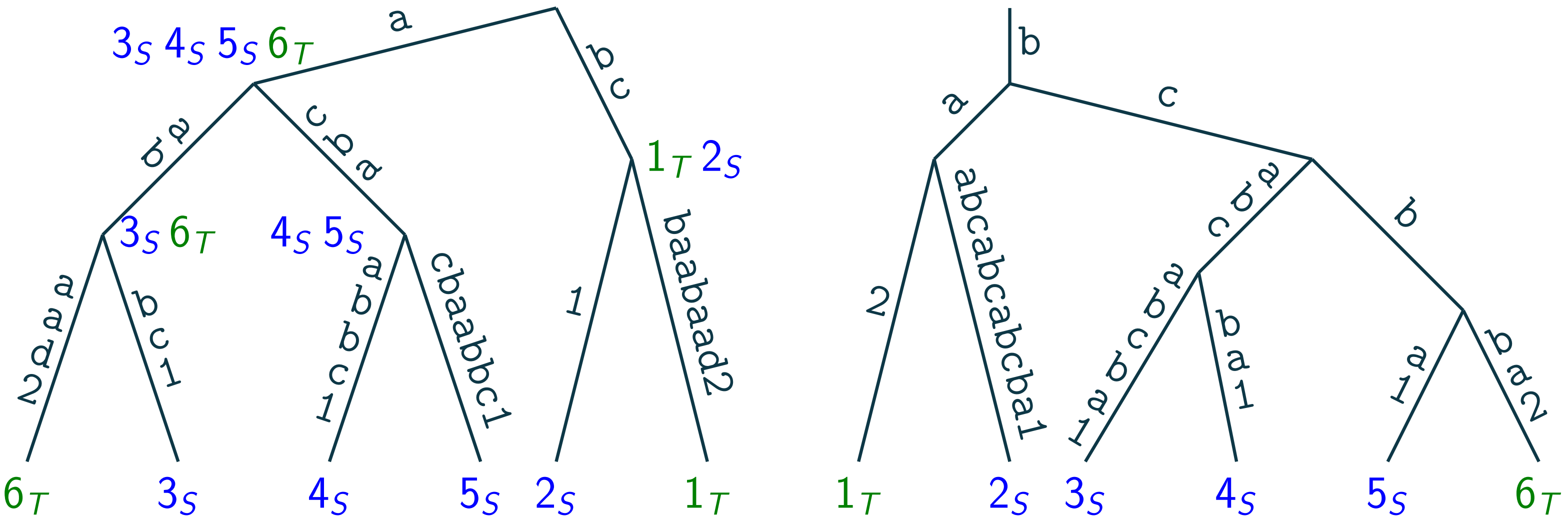

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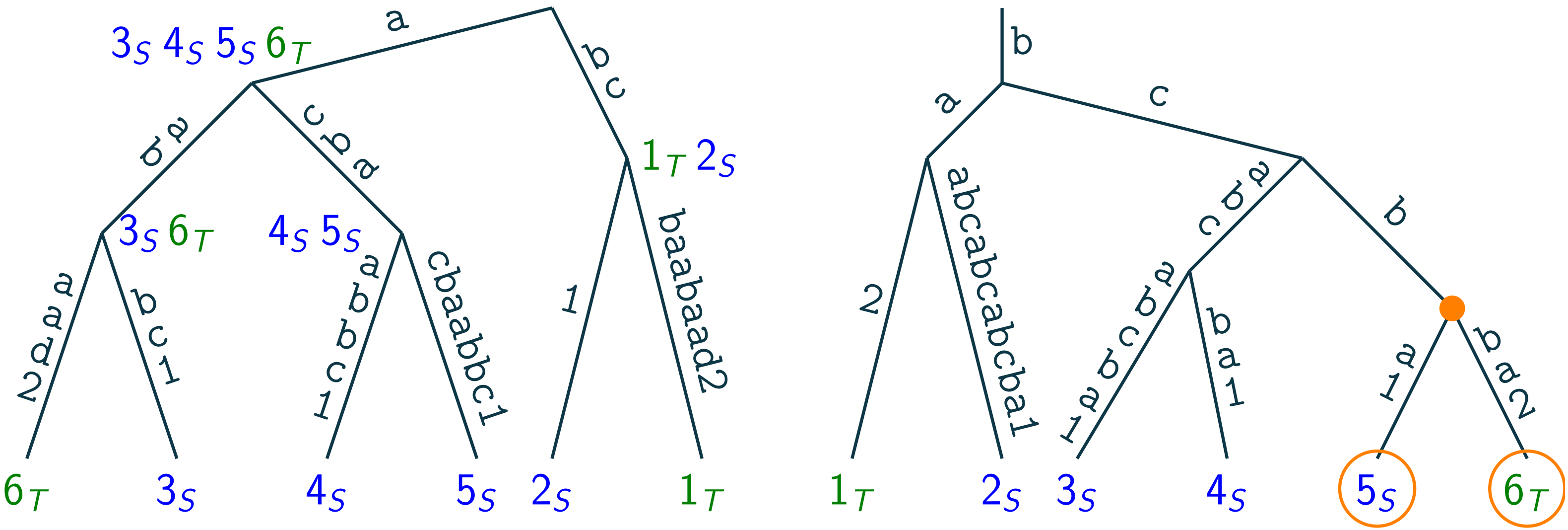

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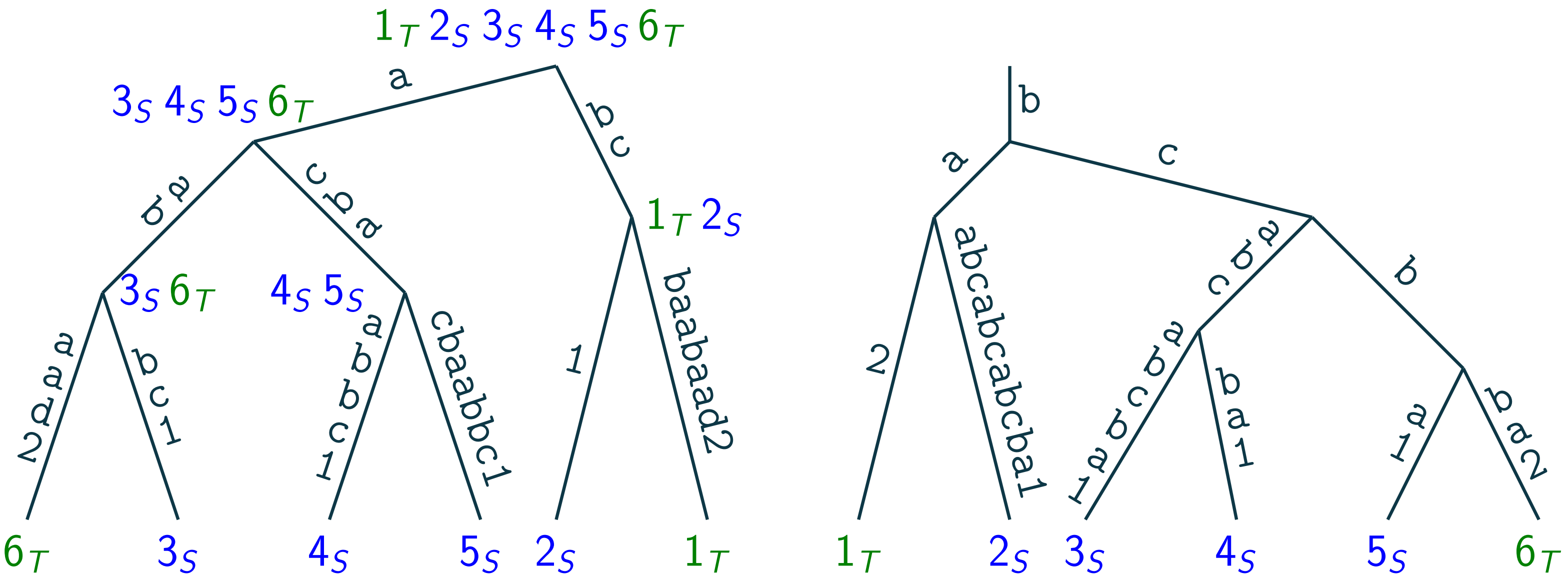

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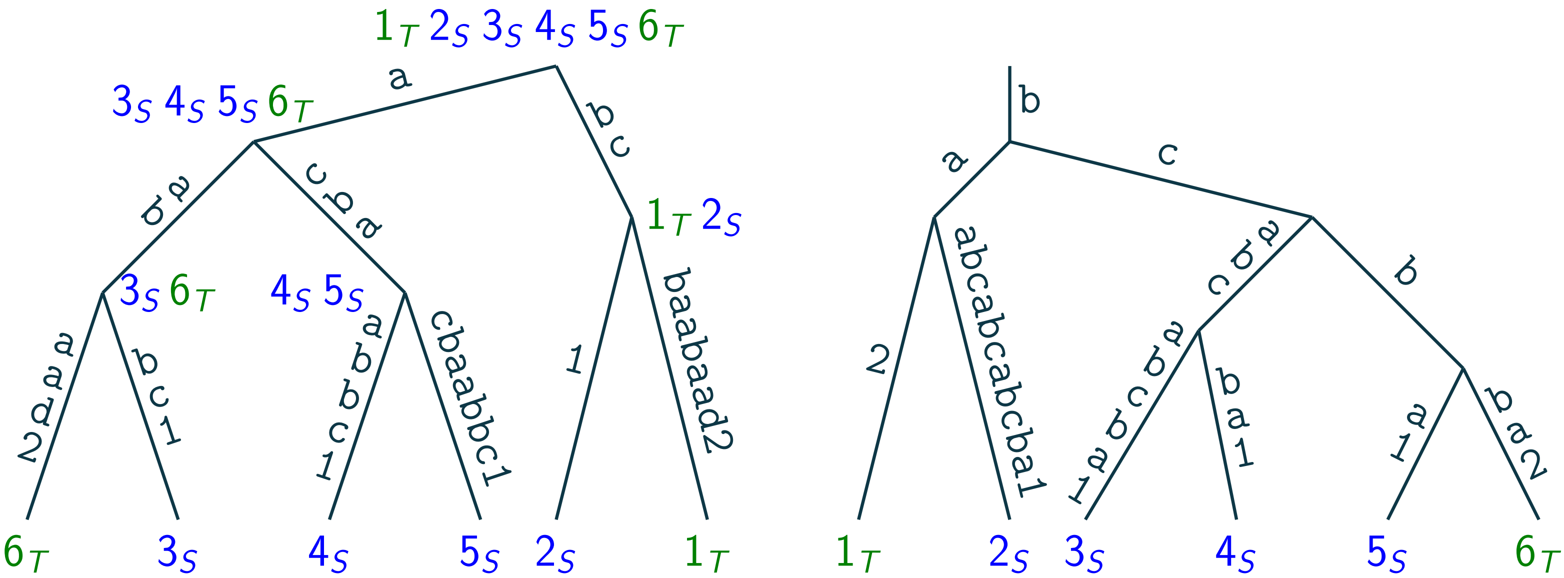

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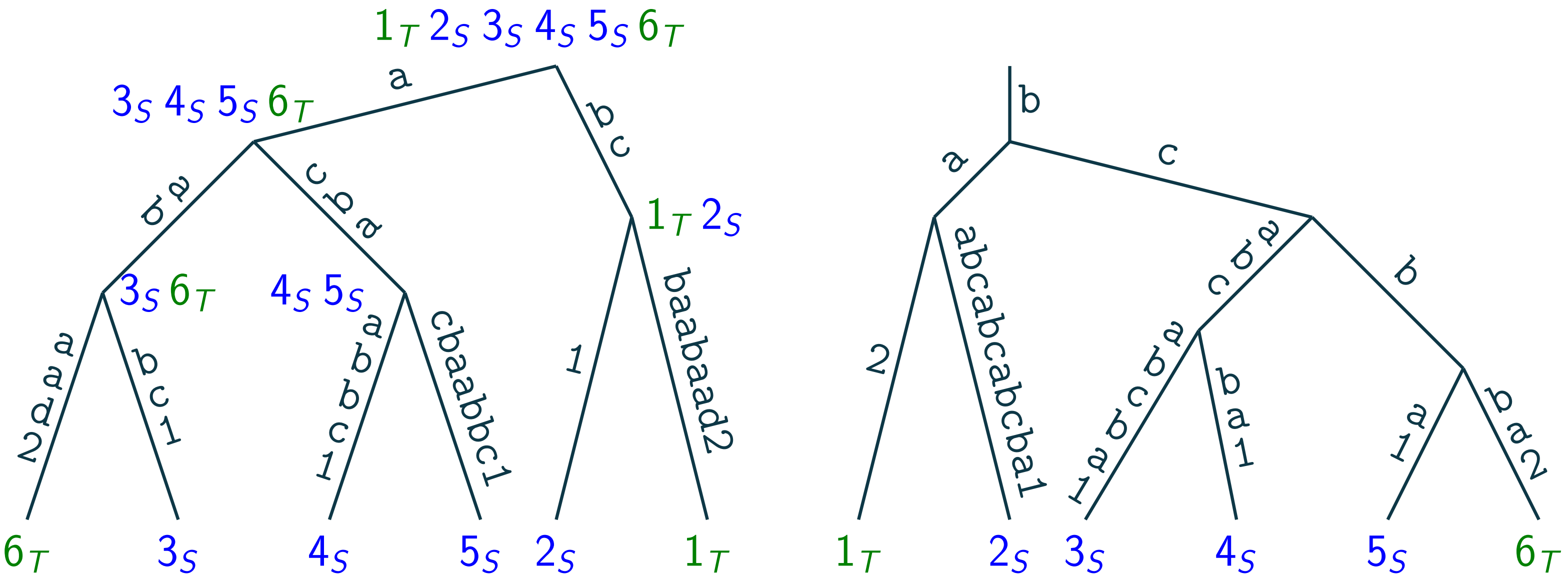
Solution to MaxPairLCP



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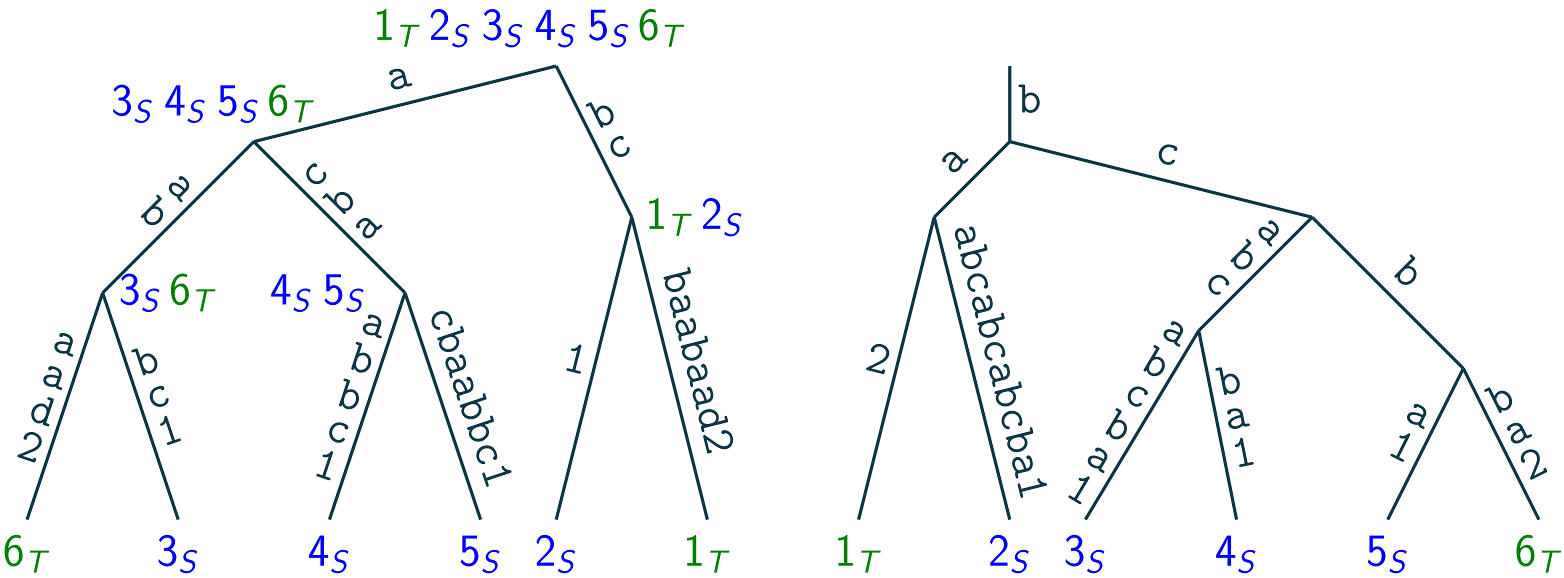
[1] M.R. Brown, R.E. Tarjan: A Fast Merging Algorithm. J. ACM, 1979

Solution to MaxPairLCP



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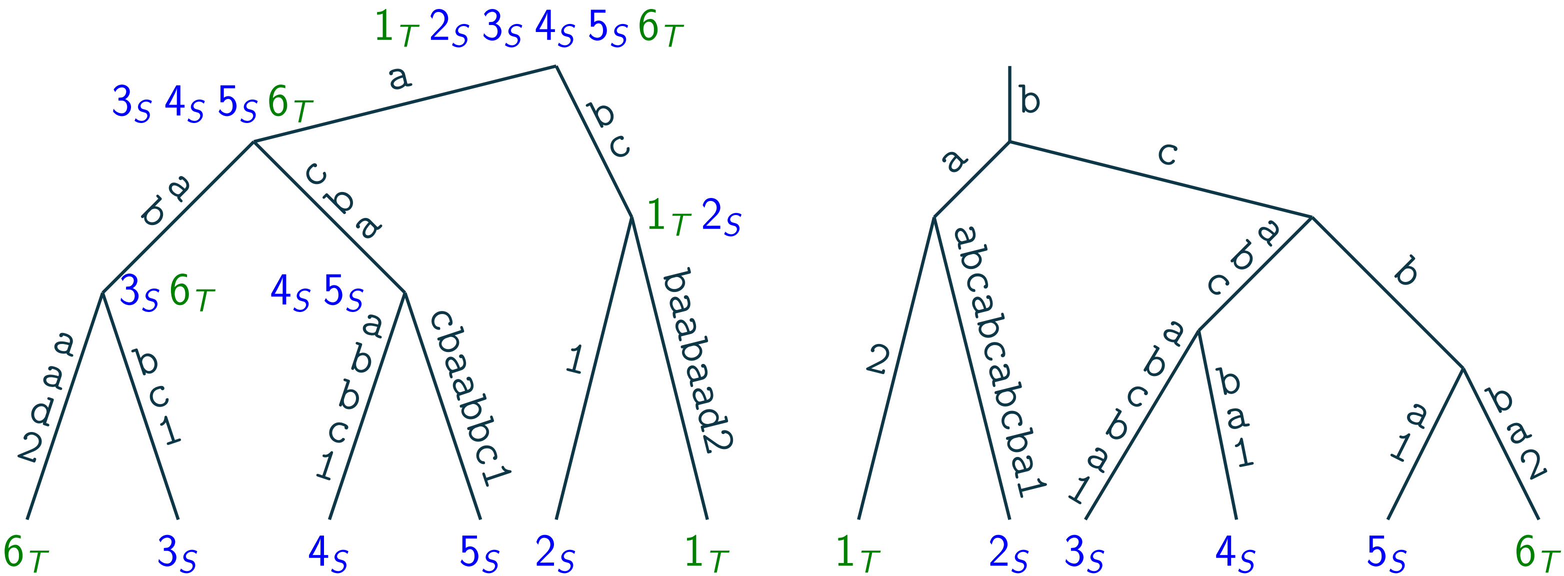
[2] M.A. Bender, M. Farach-Colton: The LCA Problem Revisited. LATIN 2000

Solution to MaxPairLCP



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MaxPairLCP in $O(N \log N + n / \log_\sigma n)$ time

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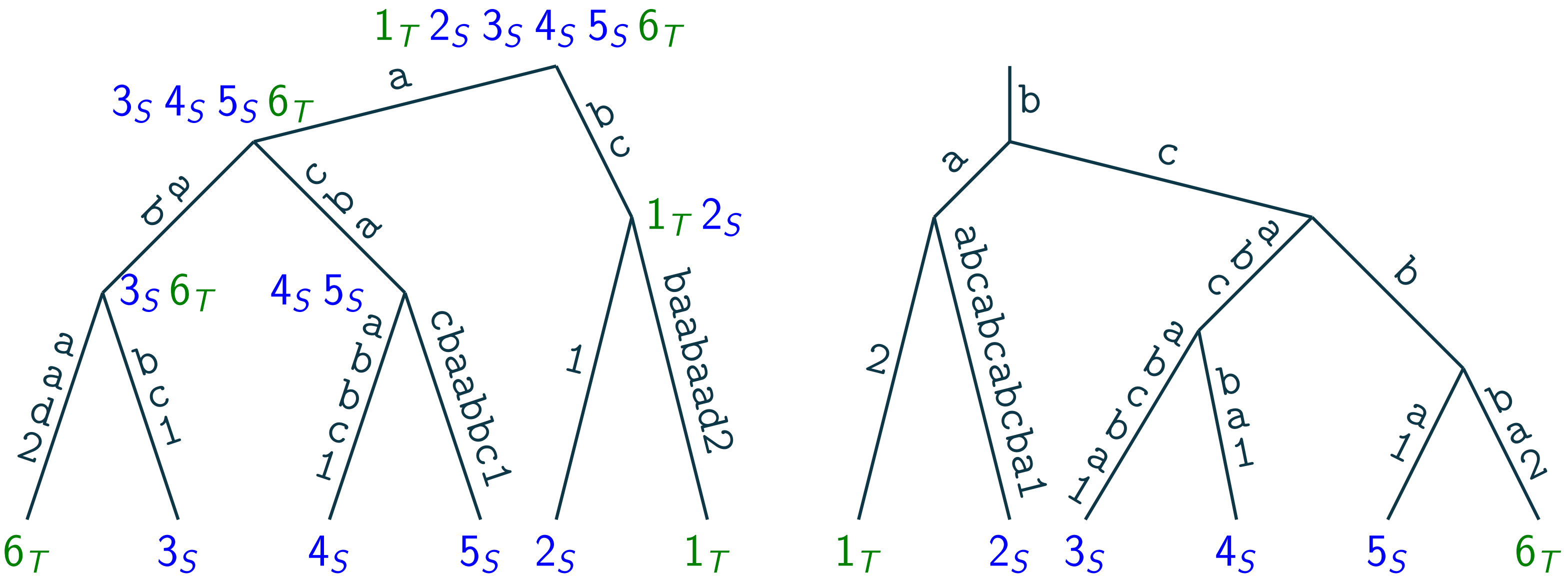

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MaxPairLCP in $O(N \log N + n/\log_\sigma n)$ time $\Rightarrow$ Long LCF in $O(n/\log n)$ time.

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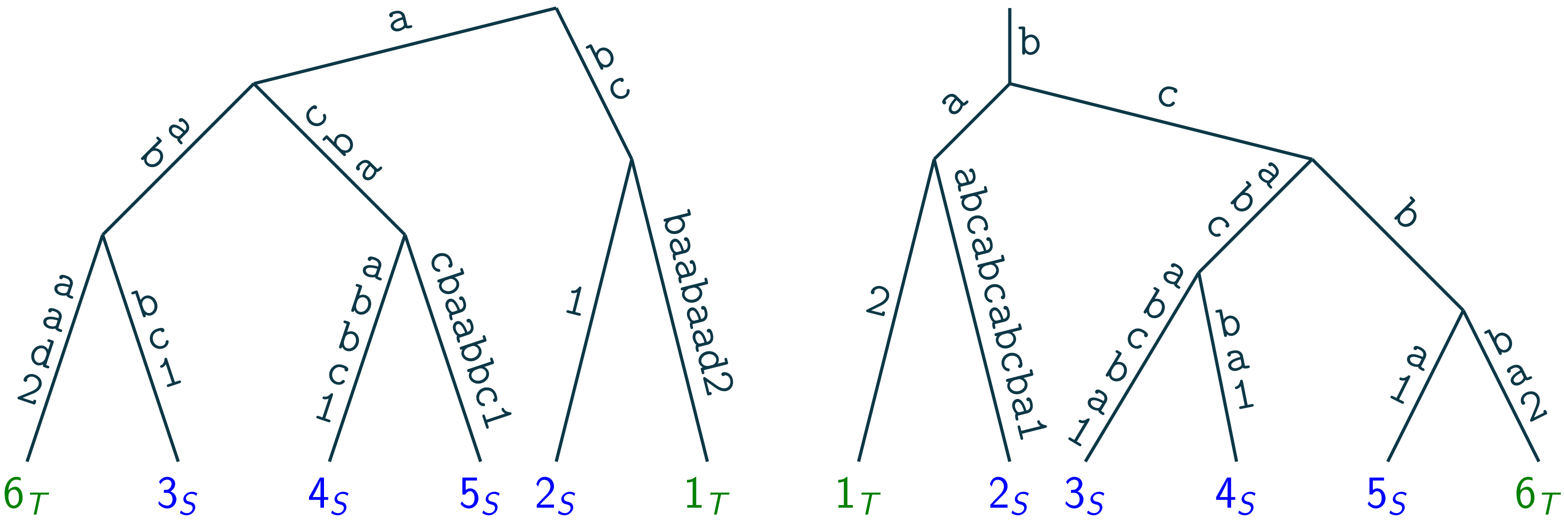
Colored Trees Problem



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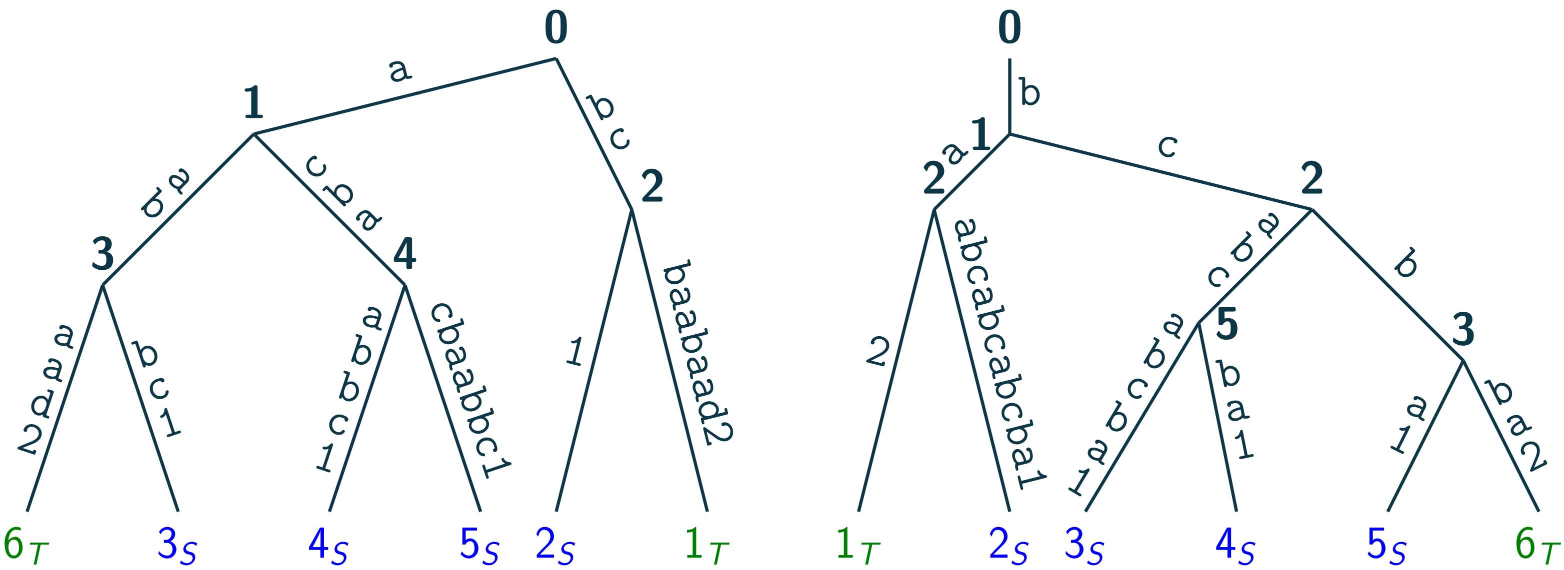

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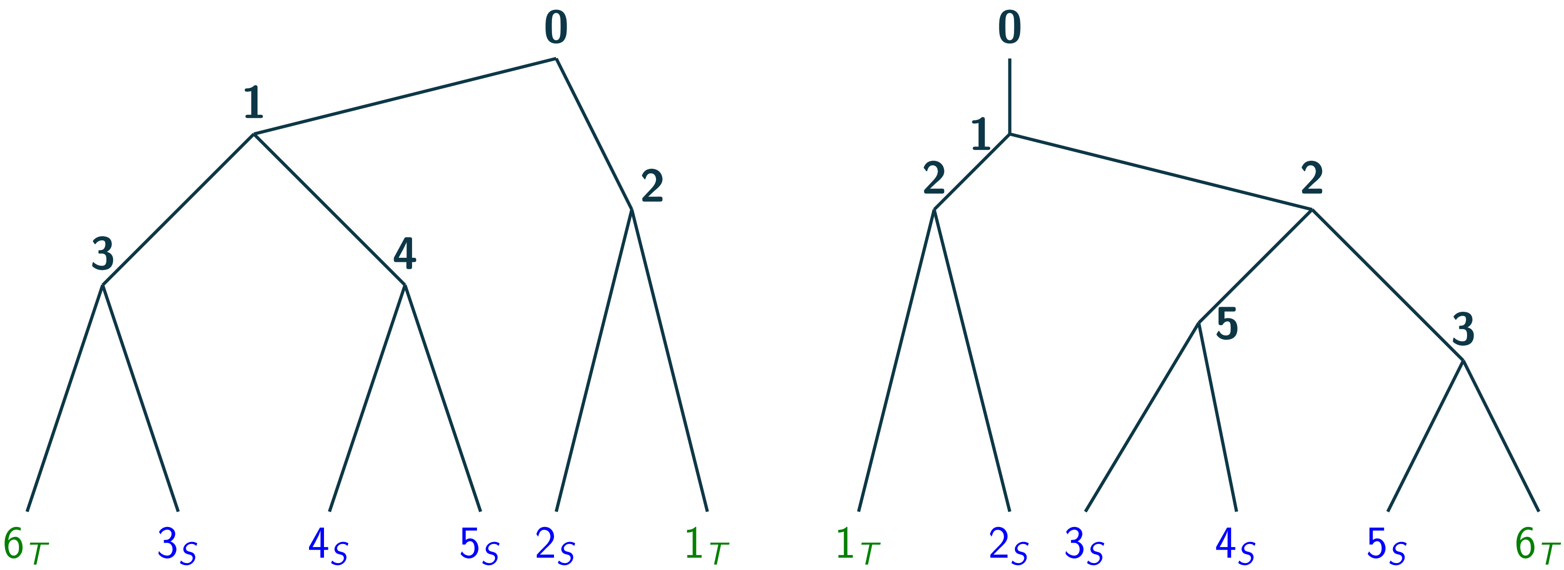
Colored Trees Problem



ColoredTrees problem

- Nodes u, v from both trees
- Their subtrees have a pair of equal leaves of different colors
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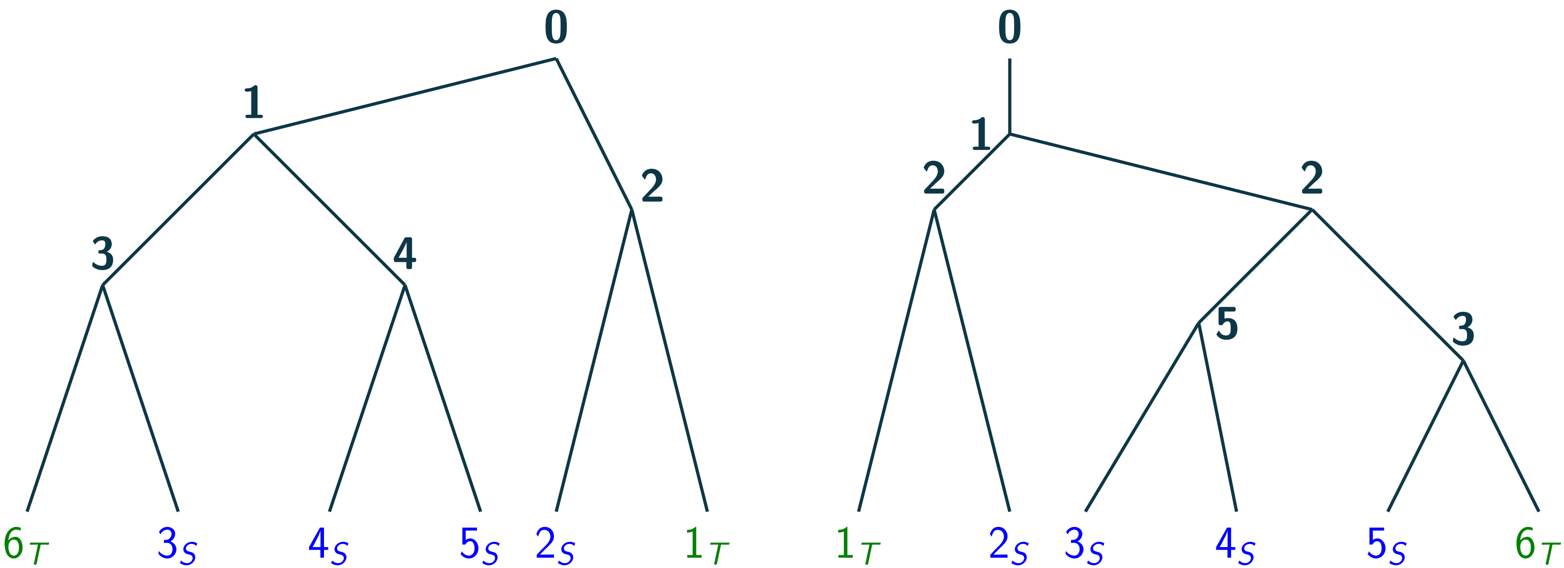

COLOREDTREES problem

- Nodes u, v from both trees
- Their subtrees have a pair of equal leaves of different colors
- $depth(u) + depth(v)$ is max

COLOREDTREES problem can be solved in $O(N \log N)$ time (N is the size of trees).

Plan of Presentation

- **Packed LCF:**
    - Short LCF
    - Long LCF
    - **Medium-length LCF**
- Approximate LCF
- Small-space LCF
- Compressed LCF
- Dynamic LCF
- Internal LCF

Medium-Length LCF

Three cases:

- Short LCF: $\leq \frac{1}{3}\log_\sigma n$: $o(n/\log n)$ time
- Medium-length LCF: $O(n/\sqrt{\log n})$ time?
- Long LCF: $\geq \log^4 n$: $O(n/\log n)$ time

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MaxPairLCP in $O(N \log N + n/\log n)$ time

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- We don't know how to beat $O(N \log N)$ in general

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MaxPairLCP in $O(N \log N + n/\log n)$ time

- We don't know how to beat $O(N \log N)$ in general
- We will consider special cases of MaxPairLCP
- and replace difference covers with string synchronizing sets [1]

[1] D. Kempa, T. Kociumaka: String Synchronizing Sets: Sublinear-time BWT Construction and Optimal LCE Data Structure. STOC 2019

String Synchronizing Set

A τ-synchronizing set of a string W of length n is a set of positions X that is:

- small: $|X| = O(n/\tau)$
- consistent: whether $i \in X$ depends only on $W[i..i+2\tau)$
- dense: $X \cap [i..i+\tau) = \emptyset$ for $i \in [1..n-3\tau+2]$
  iff $per(T[i..i+3\tau-1]) > \tau/3$
- fast to construct: in $O(n/\tau)$ time if $\tau \le \frac{1}{5}\log_\sigma n$, otherwise $O(n)$ time

a b a a b c a a a a a a a a a a b a a b c a $\qquad \tau = 3$

0 2 5 11 13 15

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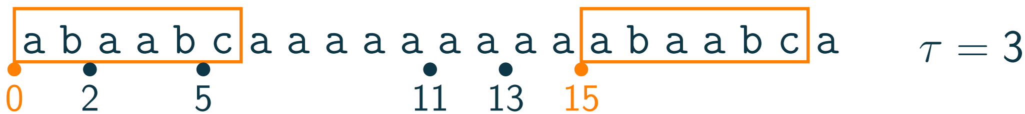

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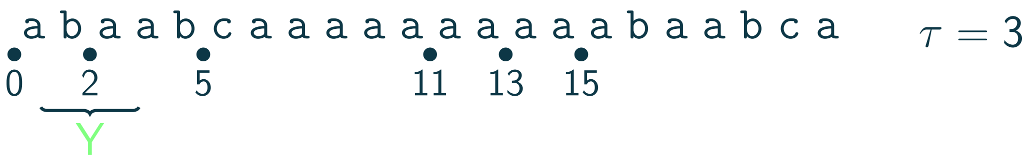

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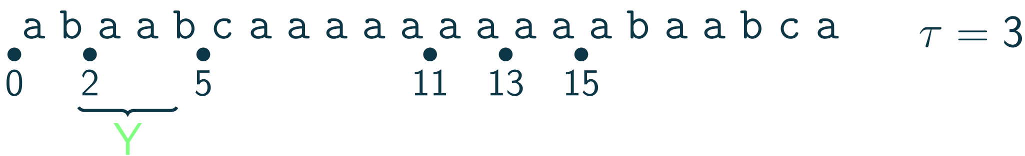

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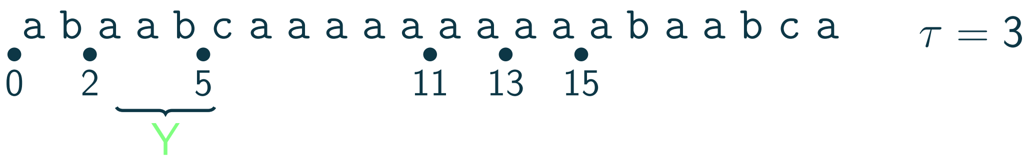

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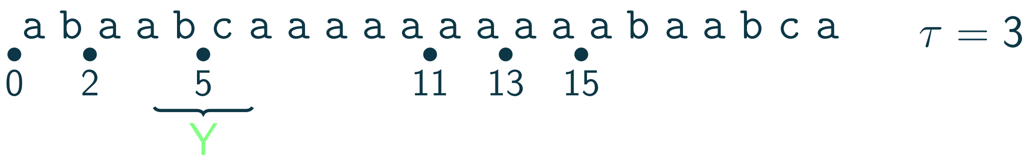

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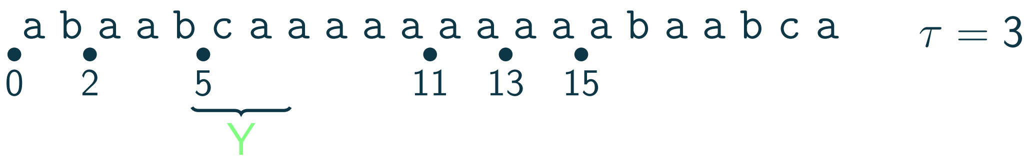

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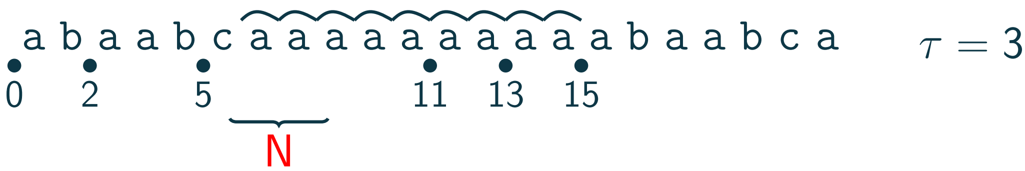

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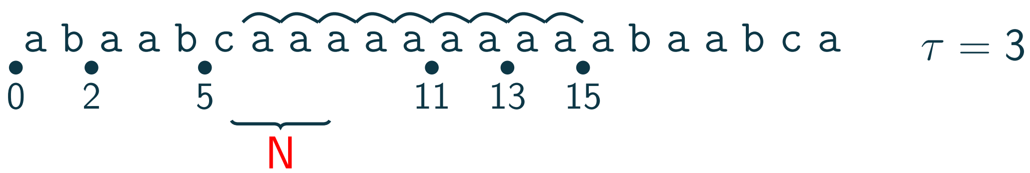
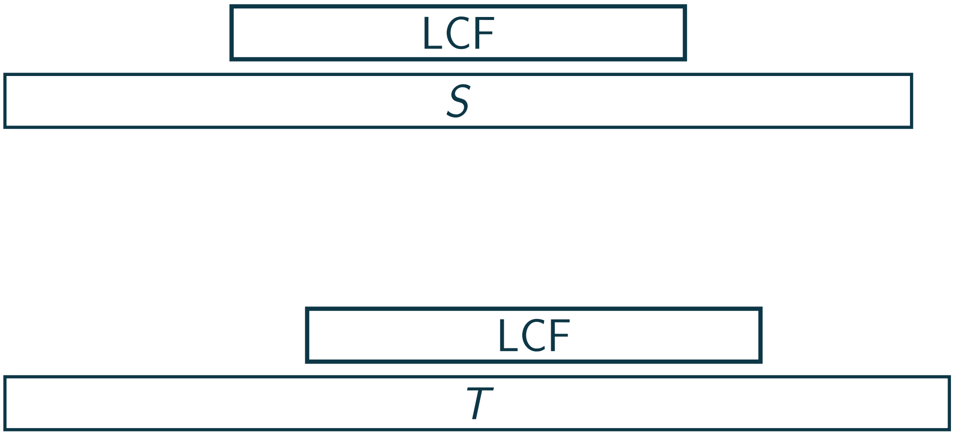


Computing Anchors

Assume the length of LCF is in $[\tau, \ell]$.



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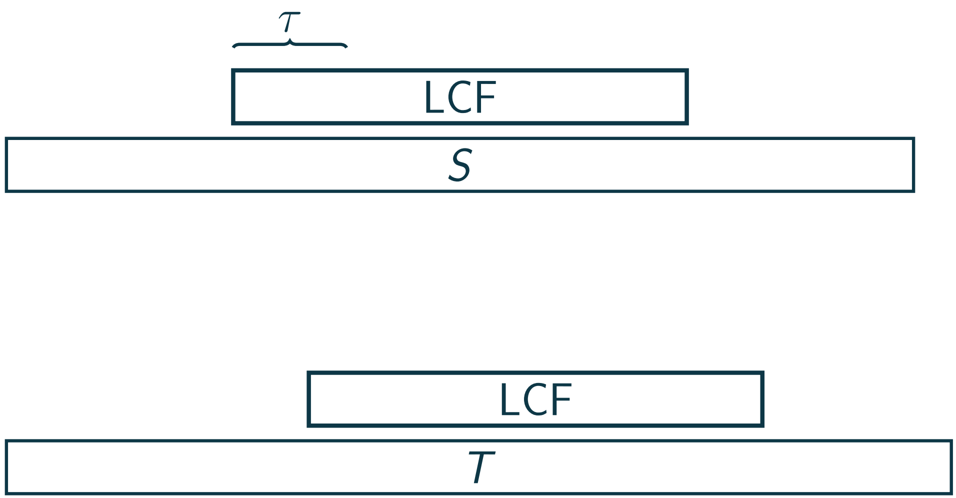

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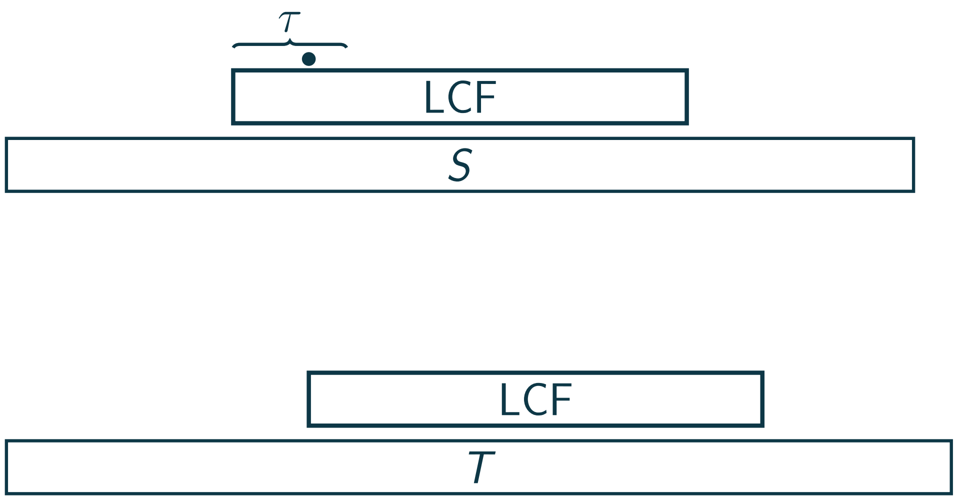

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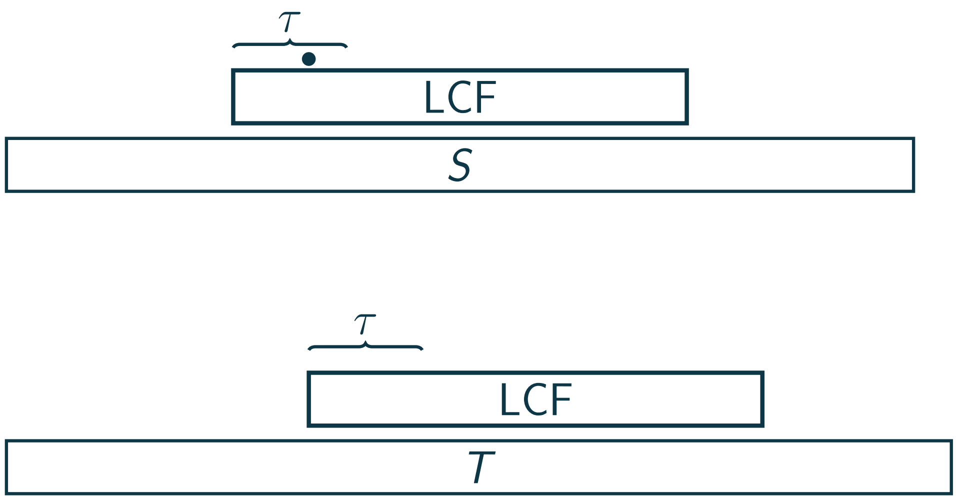

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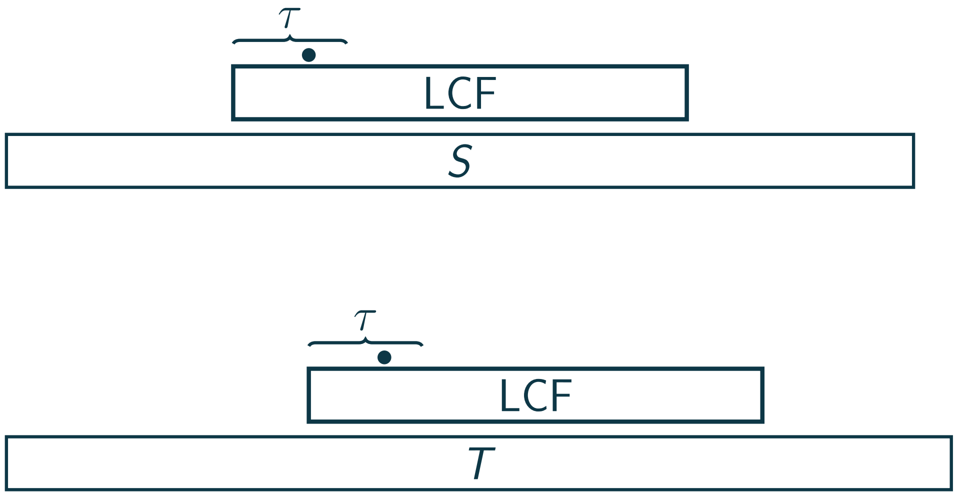

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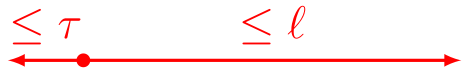

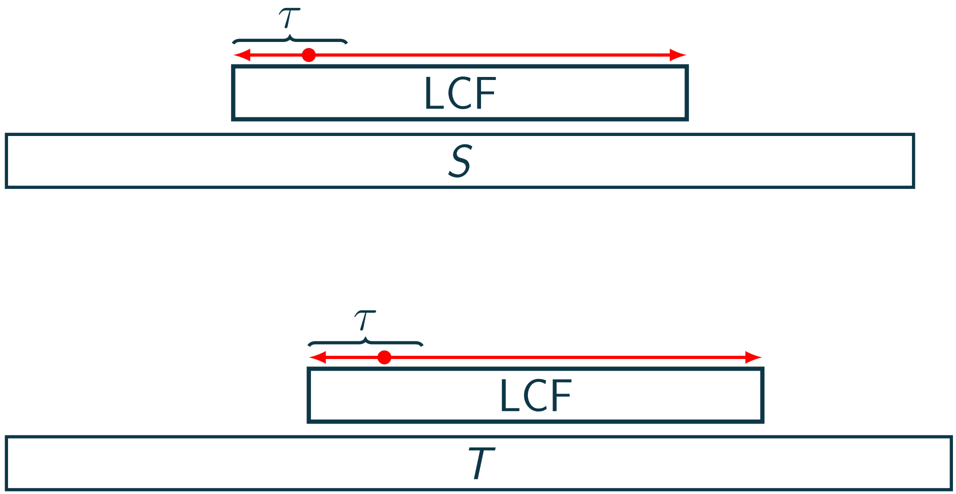

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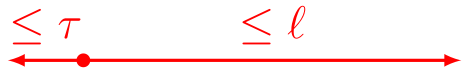

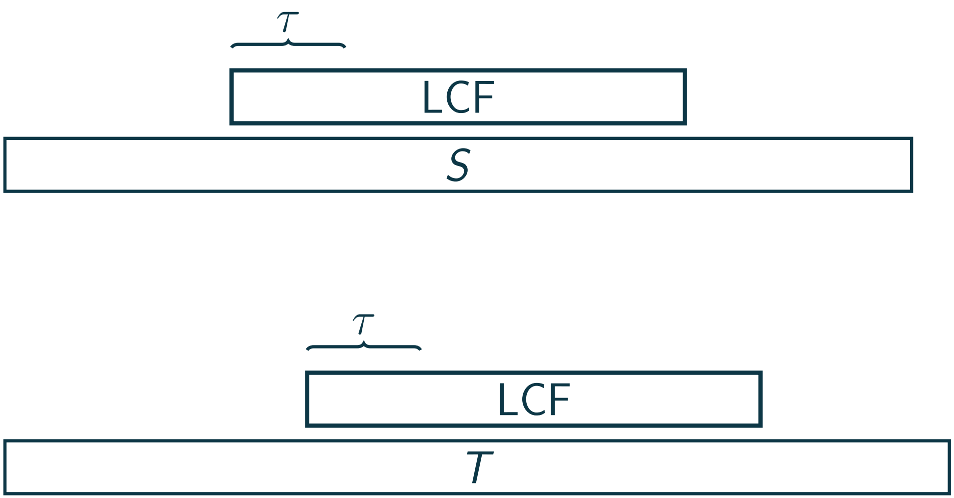

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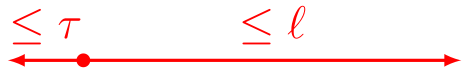

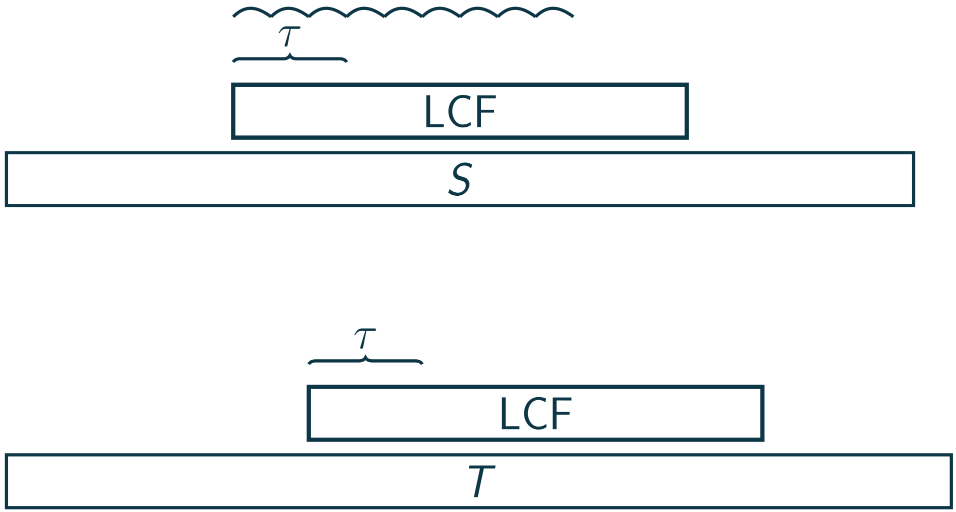

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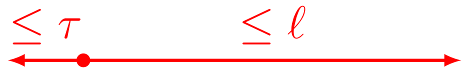

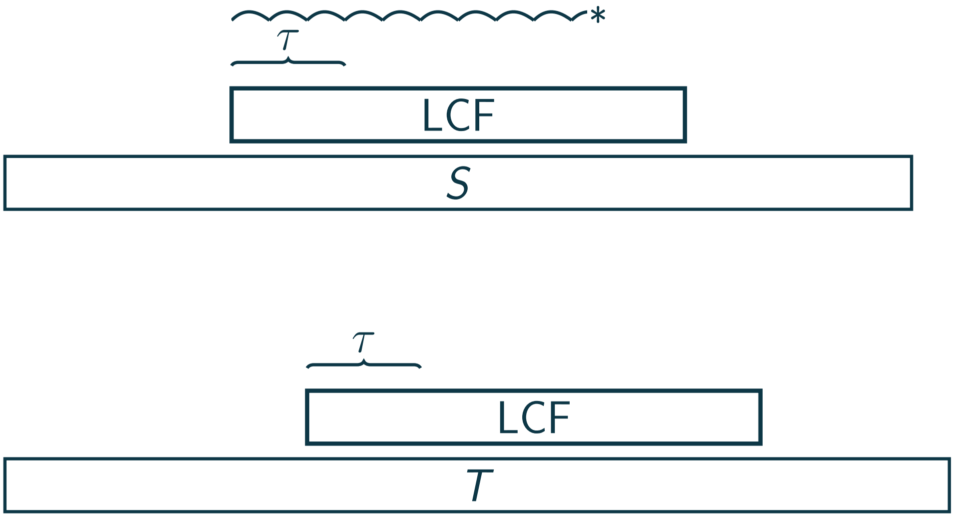

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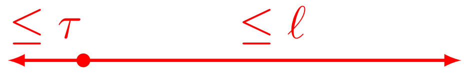

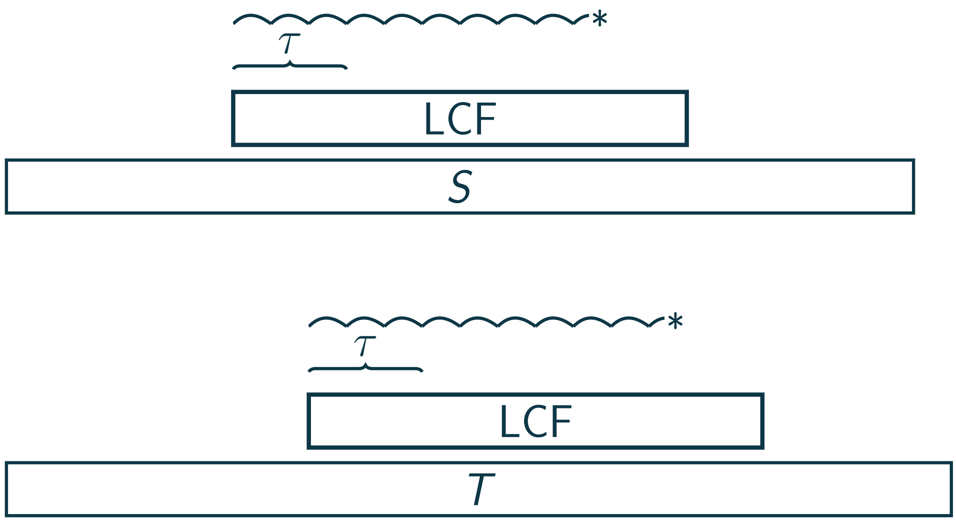

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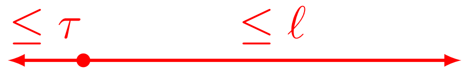

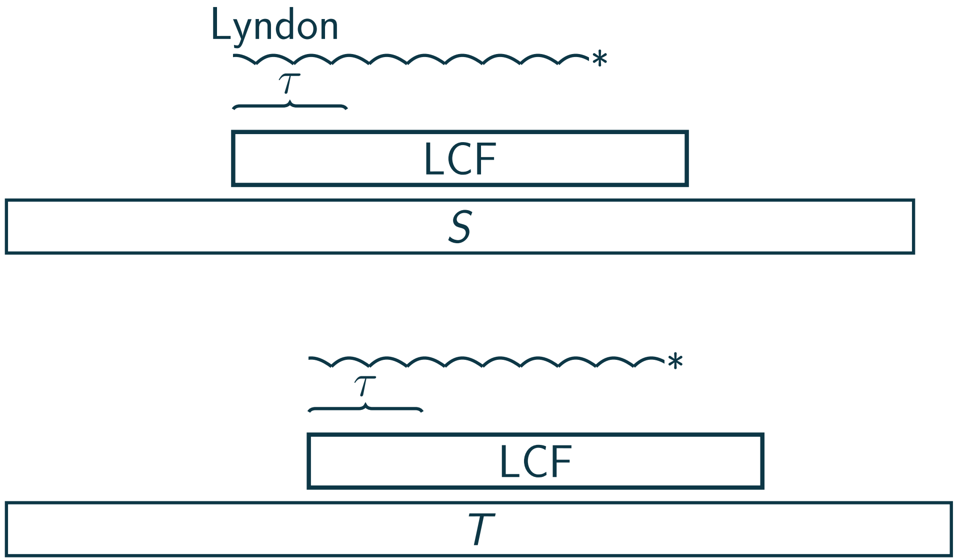

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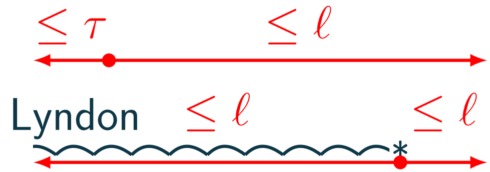

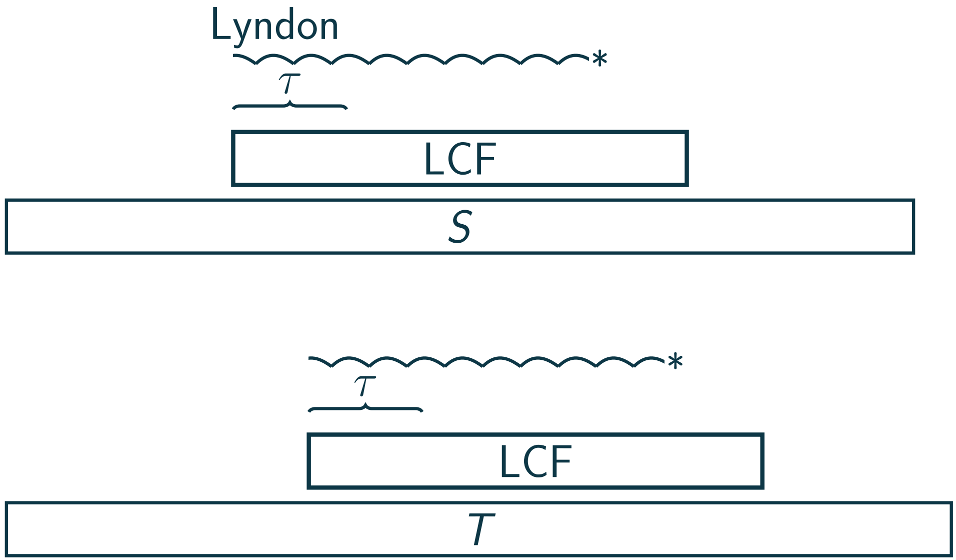

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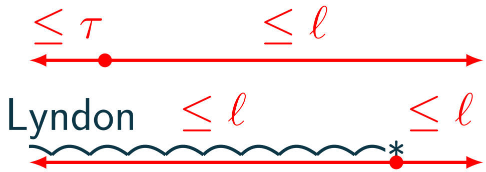

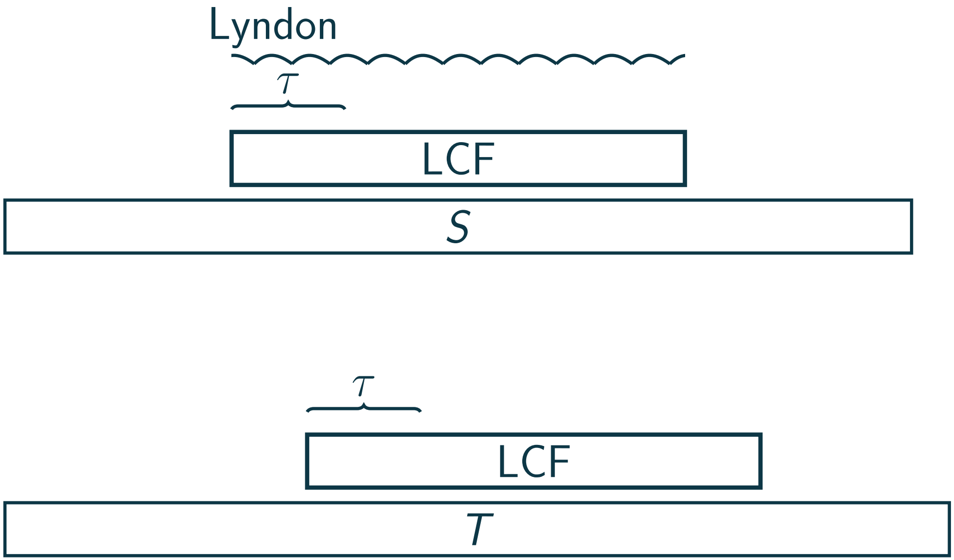

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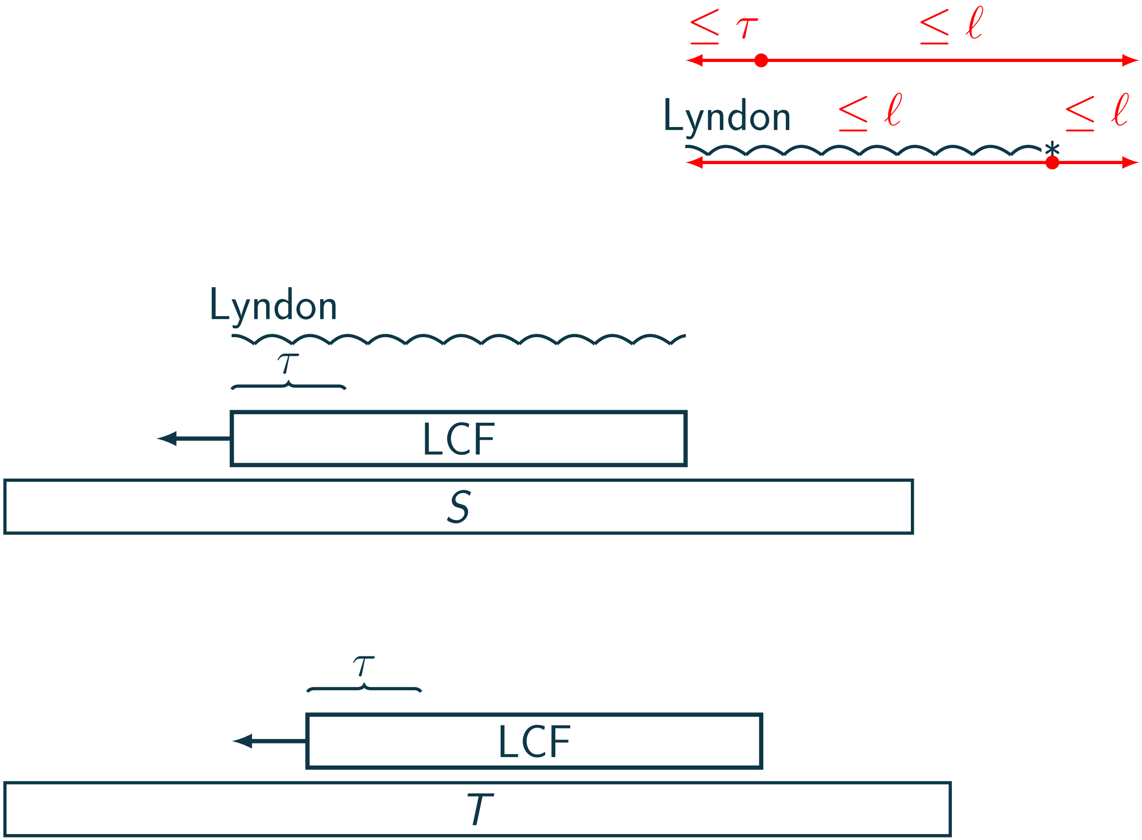

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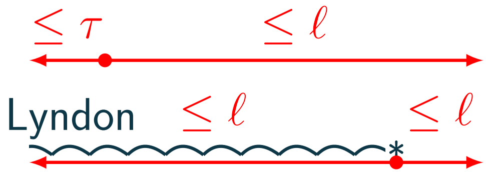

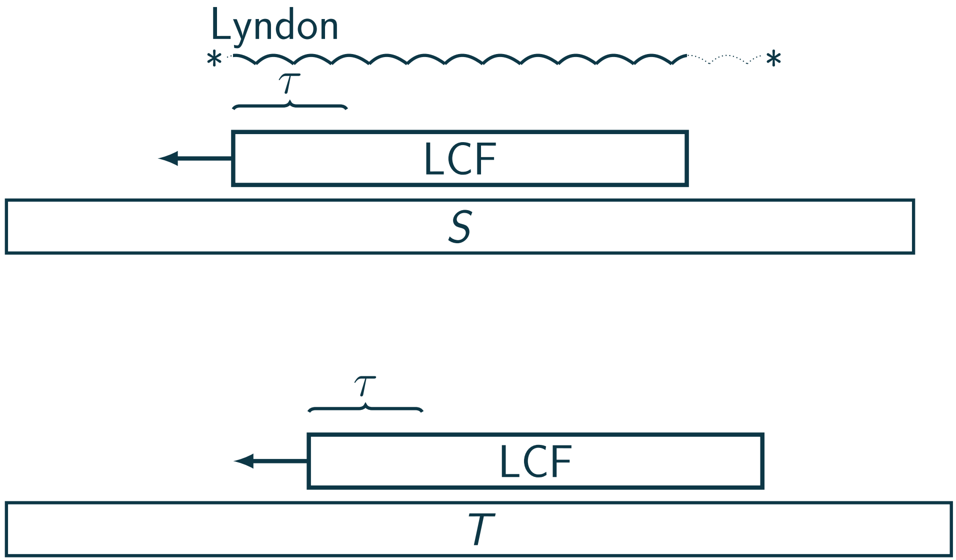

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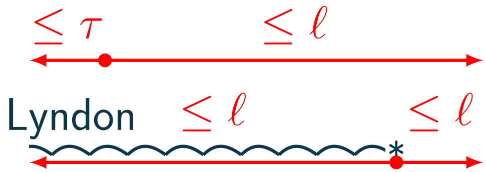

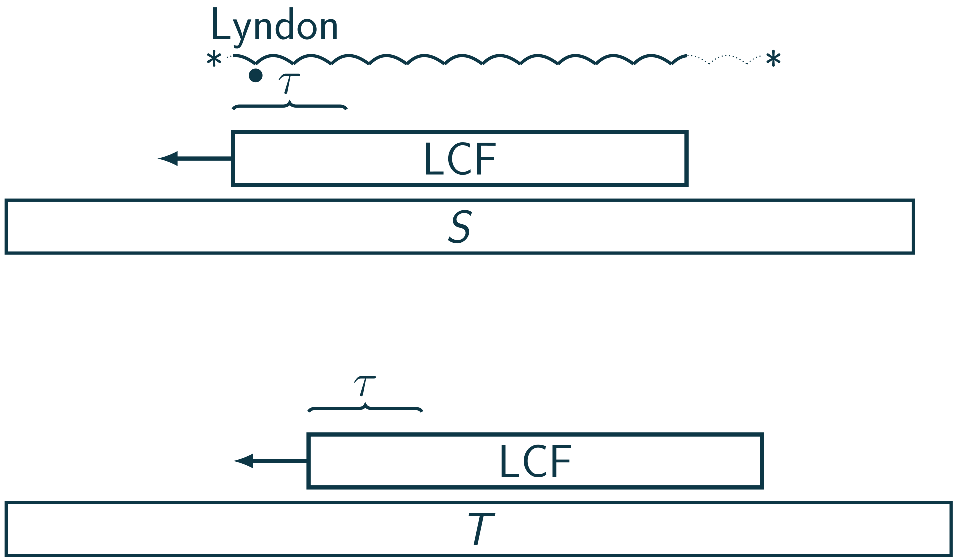

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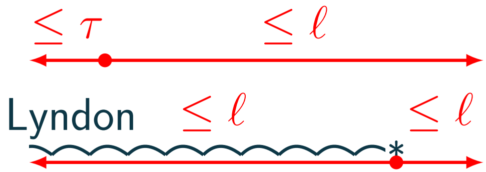

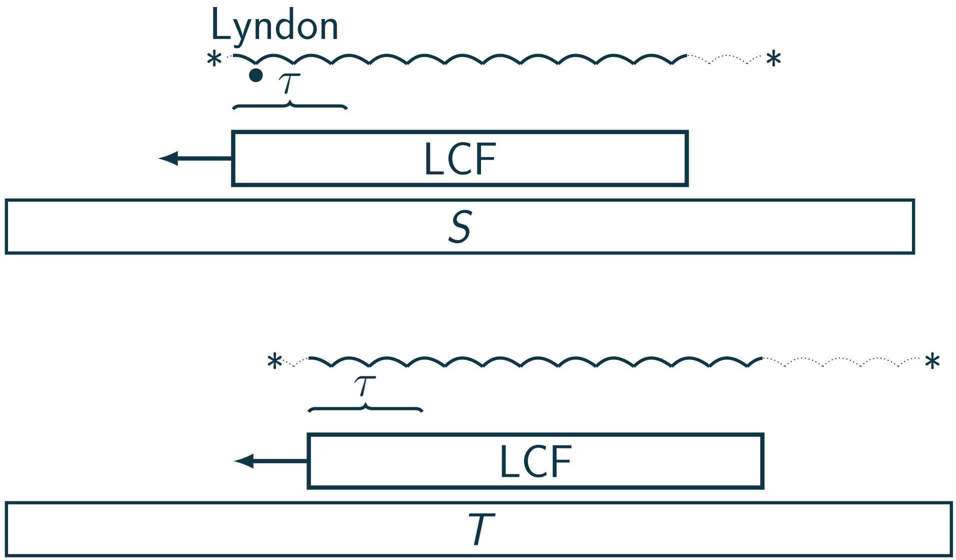

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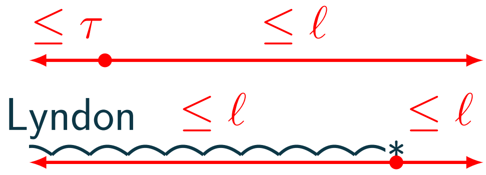

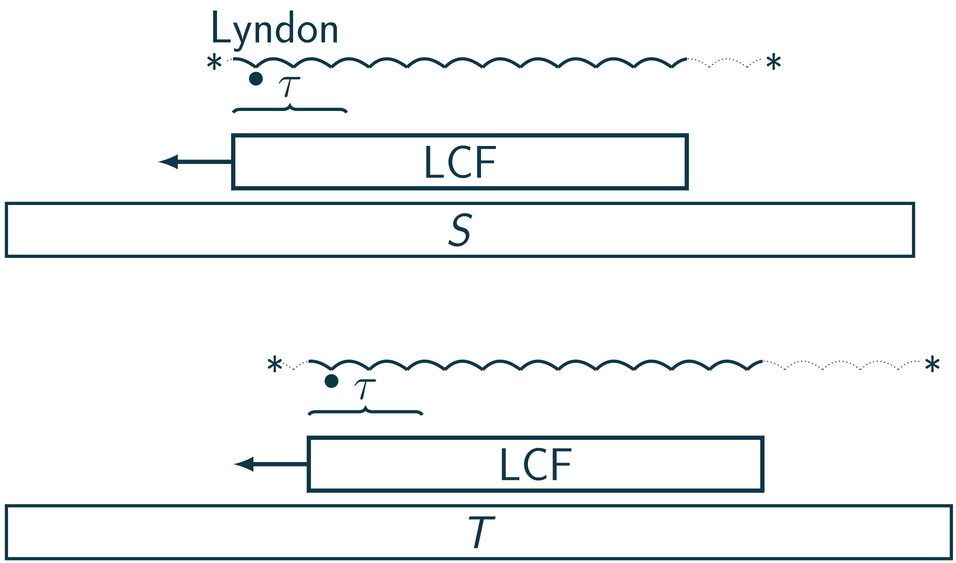

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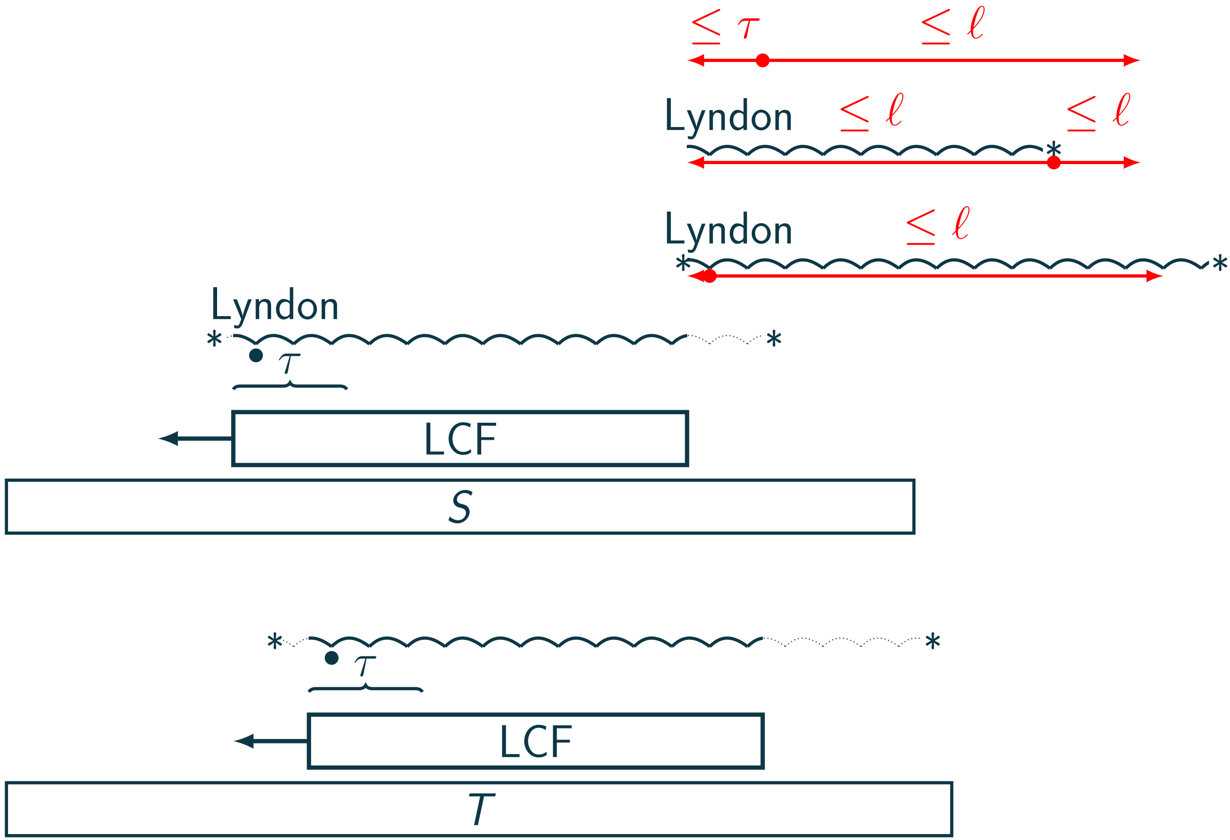

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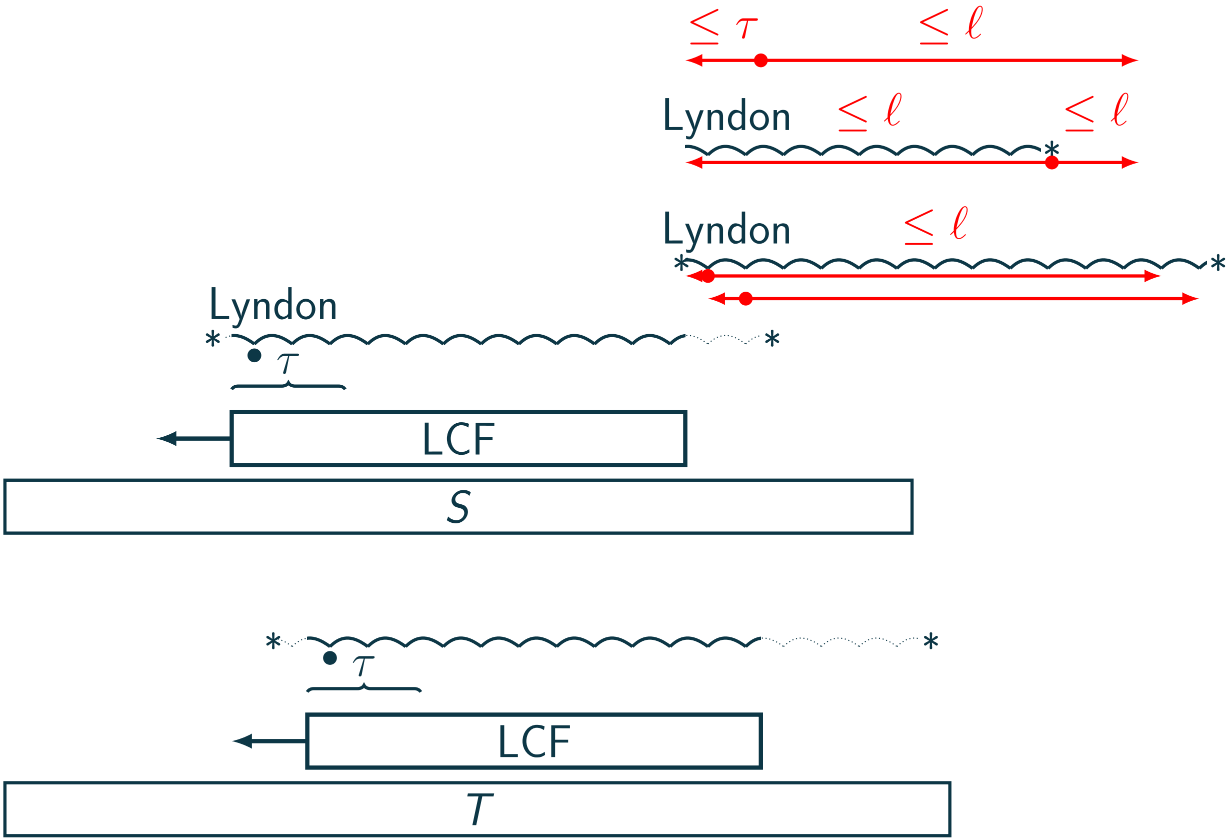

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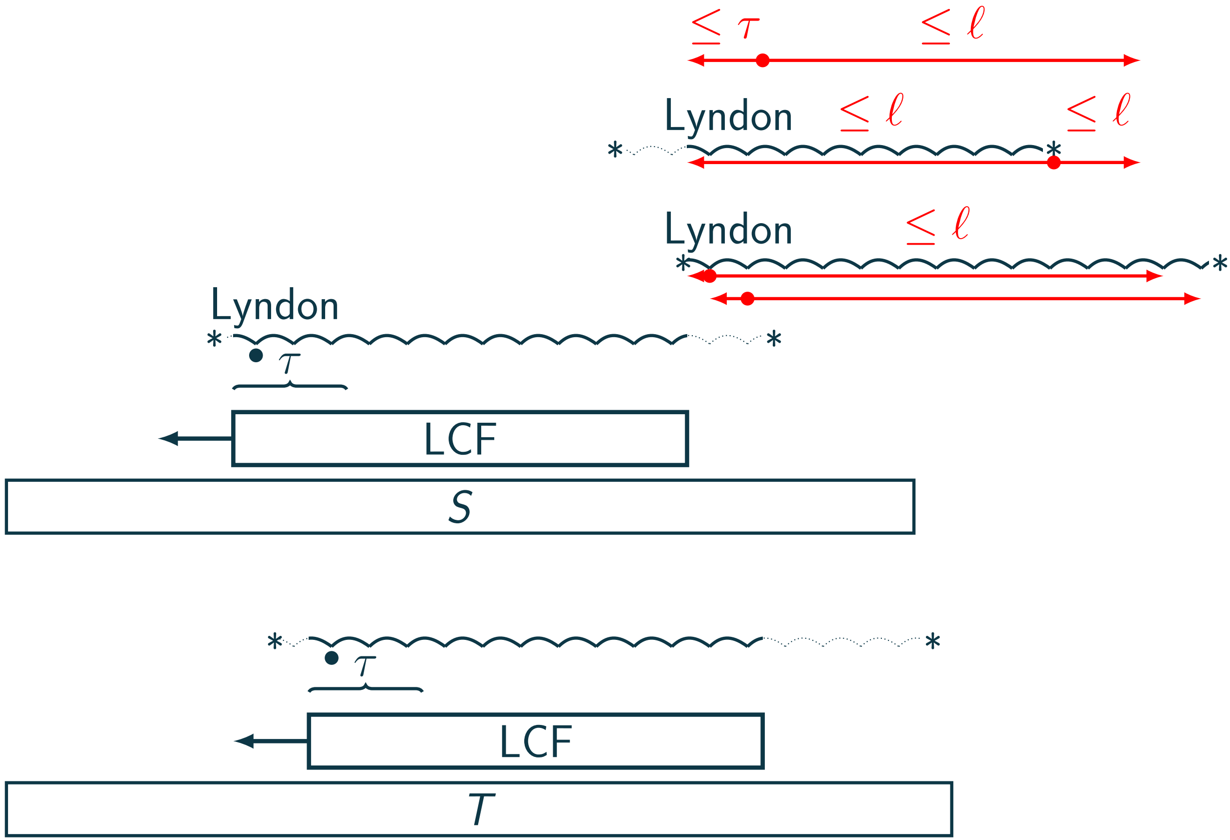


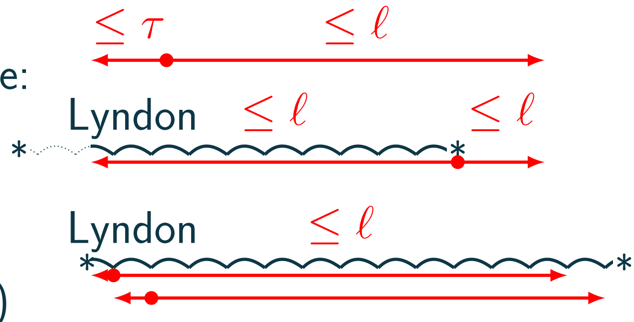
Computing Anchors

Assume the length of LCF is in $[\tau, \ell]$.

$O(n/\log n)$ anchors computed in $O(n/\log n)$ time:

- from synchronizing set
- 3 anchors from a τ-run

($\leq n/\tau$ such runs, computed in $O(n/\log n)$ time)



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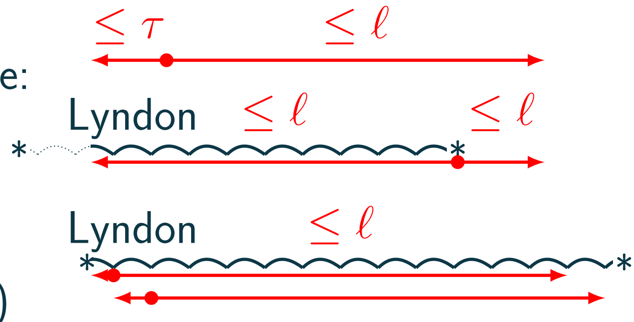

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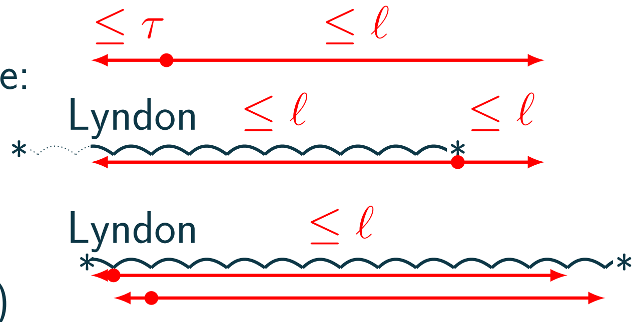

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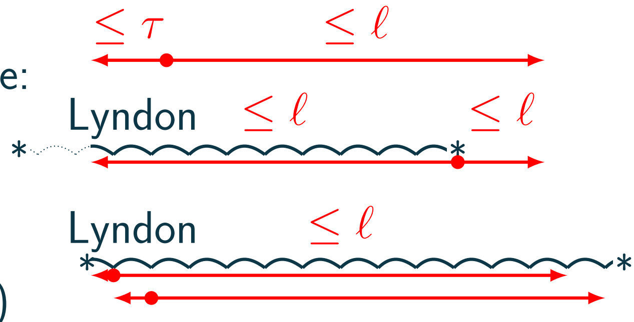

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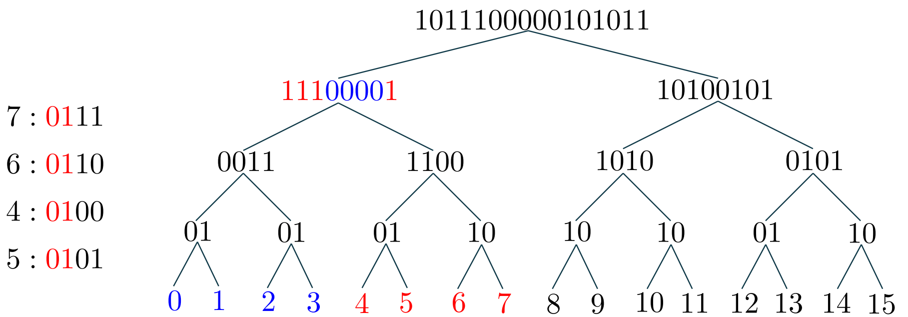
We solve type 1 using a wavelet tree.

Application of Wavelet Trees (Sketch)

Standard wavelet tree [1]:

T 12 7 11 15 9 6 4 0 1 2 10 3 13 5 8 14



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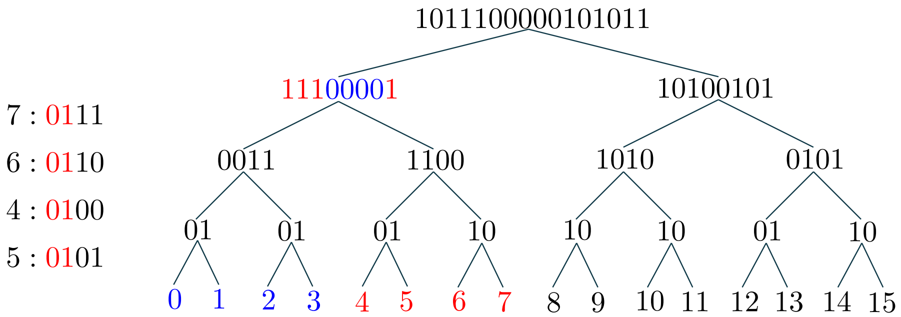

Arbitrarily shaped wavelet tree:

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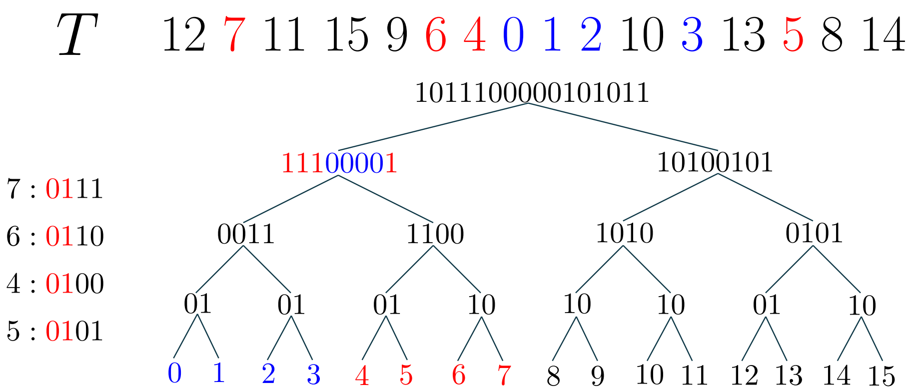

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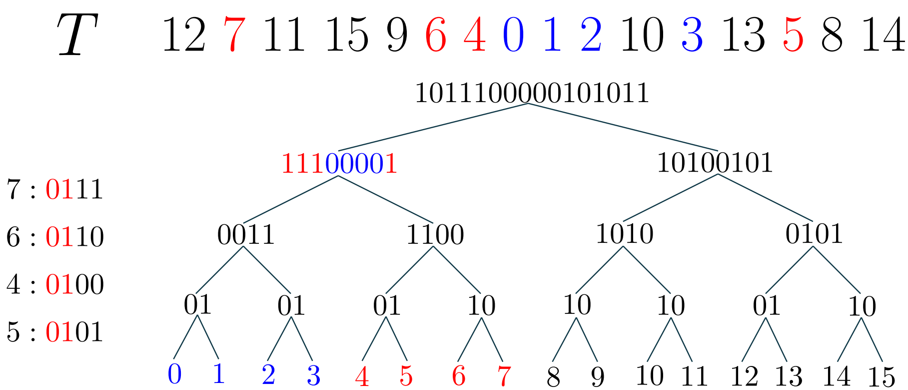

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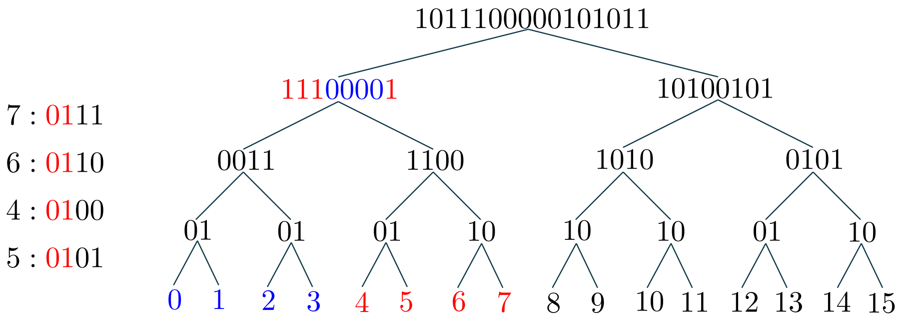

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Summary of Packed LCF

Three cases:

- Short LCF: $\ell \le \frac{1}{3}\log_\sigma n$: $o(n/\log n)$ time
- Medium-length LCF $\frac{1}{5}\log_\sigma \le \ell \le 2^{\sqrt{\log n}}$: $O(n/\sqrt{\log n})$ time
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OP: An $o(n/\sqrt{\log n})$-time algorithm via a completely different approach? [1]

T.M. Chan, M. Patrascu: Counting Inversions, Offline Orthogonal Range Counting, and Related Problems. SODA 2010

Plan of Presentation

- Packed LCF:
  - Short LCF
  - Long LCF
  - Medium-length LCF
- **Approximate LCF**
- Small-space LCF
- Compressed LCF
- Dynamic LCF
- Internal LCF

Approximate LCF, Small k

$\leq k$ mismatches allowed in the pair of substrings

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OP: Faster approximation algorithm? ([2] had $\tilde{O}(n^{3/2})$-time algorithm)

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Approximate LCF, Edit Distance

- k-edit LCF in $O(n \log^k n)$ time [1]
- $k^{1.5} n^2 / 2^{\Omega(\sqrt{(\log n)/k})}$-time for k-edit LCF [2]

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Plan of Presentation

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Small-Space LCF

- $O(s)$ space, $\tilde{O}(n^2/s)$ time for $n^{2/3} \le s \le n$ [1]
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[2] T. Kociumaka, T. Starikovskaya, and H.W. Vildhøj:
Sublinear Space Algorithms for the LCF Problem. ESA 2014

[3] S. Ben-Nun, S. Golan, T. Kociumaka, M. Kraus:
Time-Space Tradeoffs for Finding a Long Common Substring. CPM 2020

Small-Space LCF

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Goal:

- Compute LCF of compressed strings without uncompressing them.

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S and T represented as **SLPs** of size n:

- $O(n^4 \log n)$ time [1]

$|S| = N$, **LZ77** parse of S consists of n phrases, T comes as a query, $|T| = m$:

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[1] W. Matsubara, S. Inenaga, A. Ishino, A. Shinohara, T. Nakamura, K. Hashimoto: Efficient Algorithms to Compute Compressed LCFs and Compressed Palindromes. Theor. Comput. Sci., 2009

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LCF of Compressed Strings

Goal:

- Compute LCF of compressed strings without uncompressing them.

S and T represented as **SLPs** of size n:

- $O(n^4 \log n)$ time [1] $\leftarrow$ compressed overlaps and PREF table [3]

$|S| = N$, **LZ77** parse of S consists of n phrases, T comes as a query, $|T| = m$:

- $O(n \log N)$-sized data structure over S that computes LCF of S and T in $\tilde{O}(m)$ time w.h.p. [2] $\leftarrow$ Heaviest Induced Ancestor queries

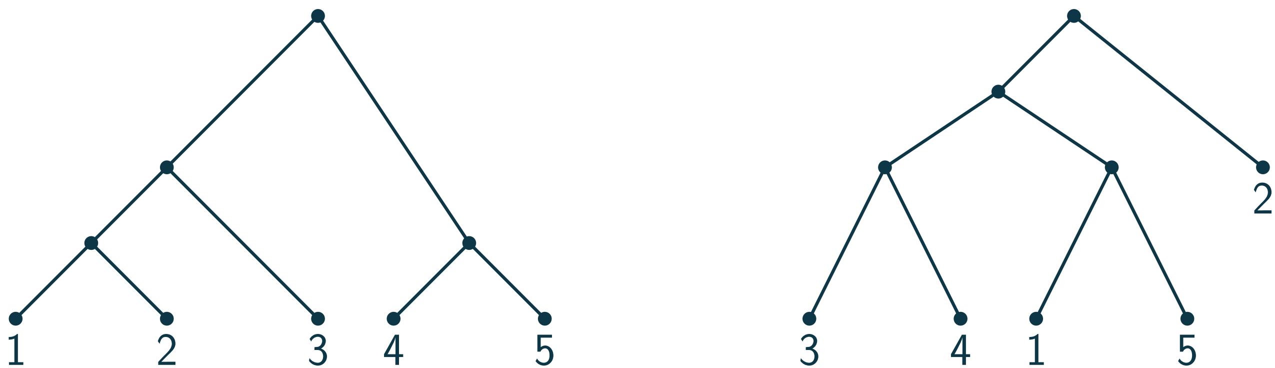
[1] W. Matsubara, S. Inenaga, A. Ishino, A. Shinohara, T. Nakamura, K. Hashimoto: Efficient Algorithms to Compute Compressed LCFs and Compressed Palindromes. Theor. Comput. Sci., 2009

[2] T. Gagie, P. Gawrychowski, Y. Nekrich: Heaviest Induced Ancestors and LCFs. CCCG 2013

[3] M. Karpinski, W. Rytter, A. Shinohara: An Efficient Pattern-Matching Algorithm for Strings with Short Descriptions. Nord. J. Comput., 1997

Heaviest Induced Ancestor (HIA) Queries

- Two trees on same set of leaves $\{1, \ldots, N\}$



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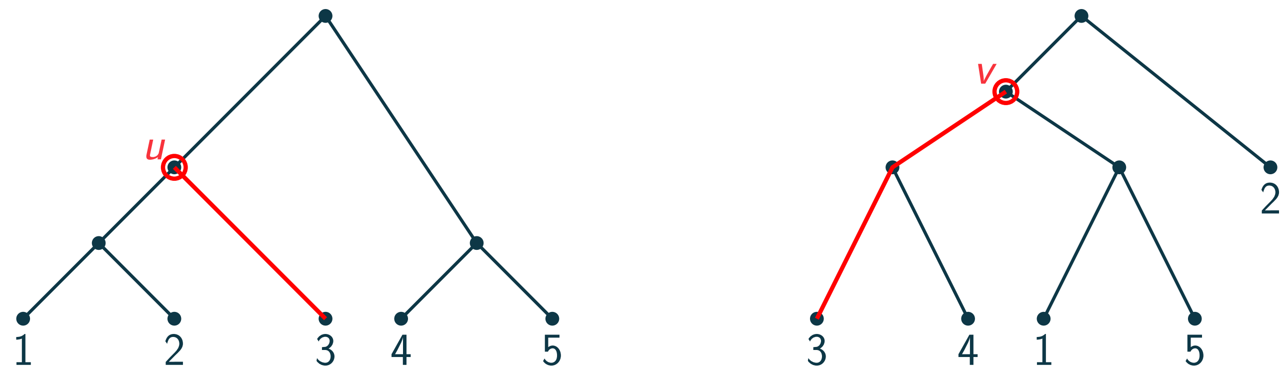
- Two trees on same set of leaves $\{1, \ldots, N\}$
- Pair of nodes is induced if their subtrees contain a common leaf



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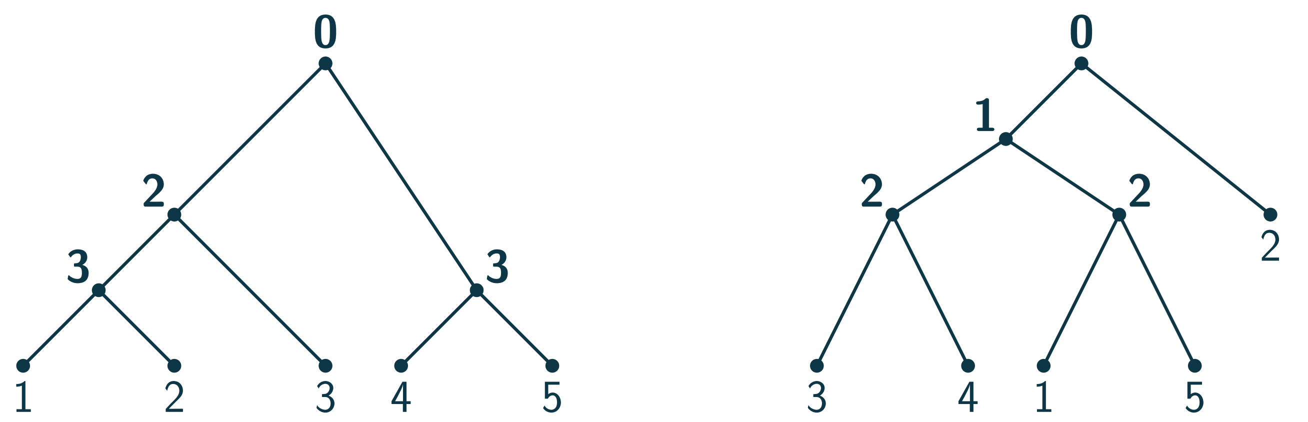

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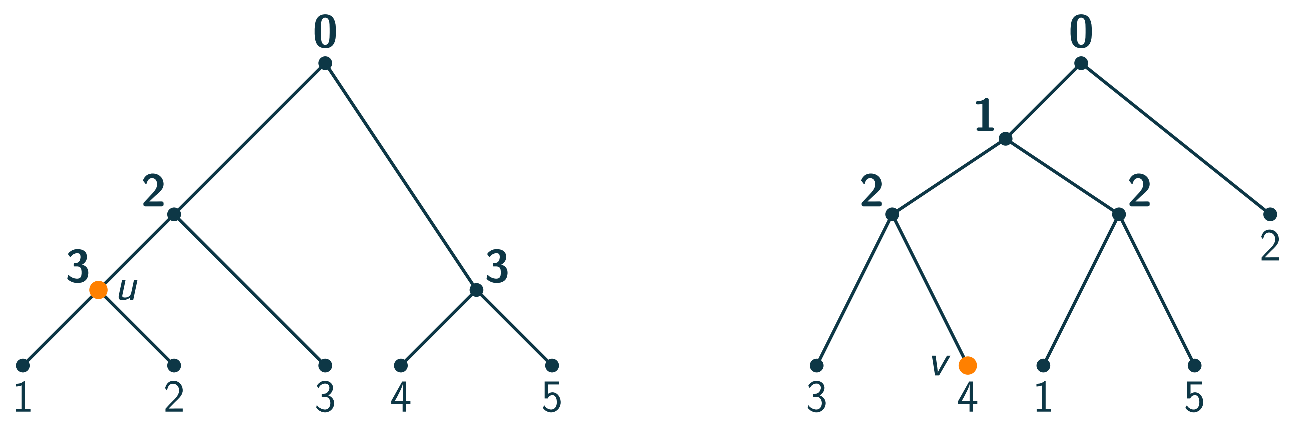

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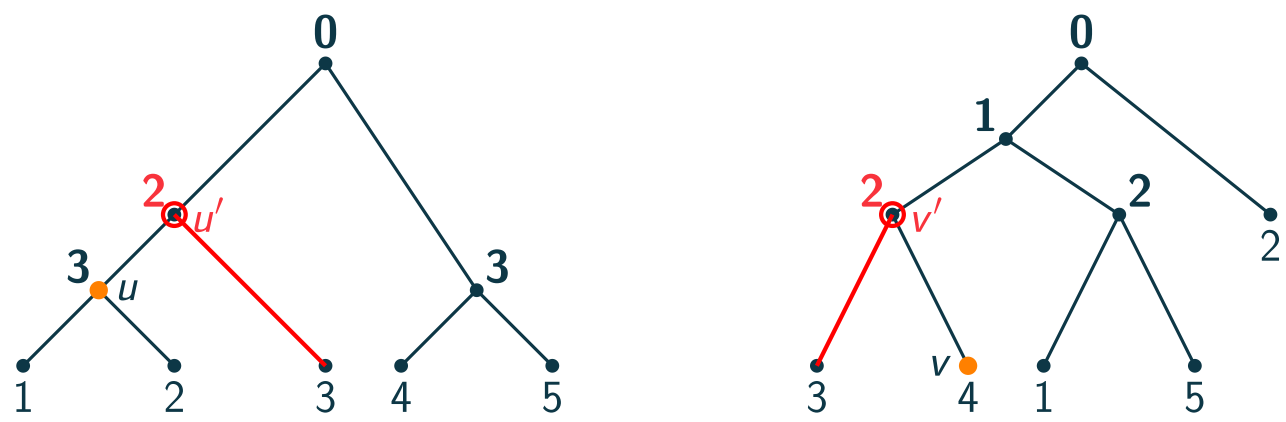

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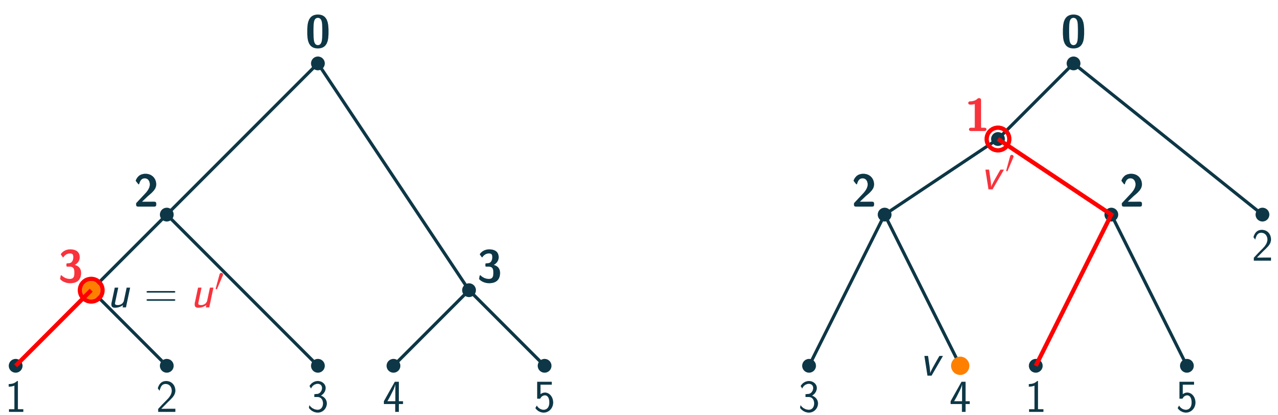

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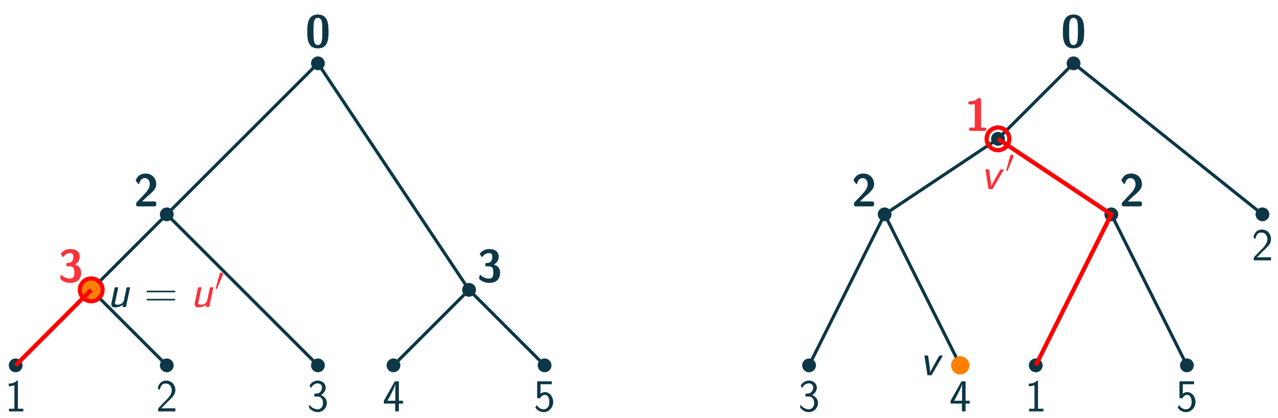
$\textsc{ColoredTrees} = N$ HIA queries



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$\textsc{ColoredTrees} = N$ HIA queries

- Various trade-offs for HIA queries given in [1-4], lower bound proved in [4]

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Heaviest Induced Ancestor (HIA) Queries

size	query	paper
$\tilde{O}(N)$	$\Omega(\log N/\log\log N)$	[4]
$O(N)$	$O(\log^2 N/\log\log N)$	[2]
$O(N\log N)$	$O(\log N\log\log N)$	[2]
$O(N\log^2 N)$	$O(\log N)$	[1]
$O(N\log^{2+\epsilon} N)$	$O(\log N/\log\log N)$	[3]
$O(N^{1+\epsilon})$	$O(1)$	[4]

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OP: Better trade-offs?

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Plan of Presentation

- Packed LCF:
  - Short LCF
  - Long LCF
  - Medium-length LCF
- Approximate LCF
- Small-space LCF
- Compressed LCF
- **Dynamic LCF**
- Internal LCF

Dynamic LCF

- input strings keep changing: insertions, deletions, substitutions allowed
- need to update the LCF after each operation

c b b a a b c a b c a b c b a

d a a b a a b c b b a

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c b b a a b c a b a a b c b a

d a a b a c a b c b b a

- $\log^{O(1)} n$-time "after-edit" queries with $\tilde{O}(n)$-sized data structure [1]
- fully dynamic $\tilde{O}(n^{2/3})$-time updates [2]
- fully dynamic $\log^{O(1)} n$ amortized time substitutions [3]

[1] A. Amir, P. Charalampopoulos, C.S. Iliopoulos, S.P. Pissis, **R**:
LCF After One Edit Operation. SPIRE 2017

[2] A. Amir, P. Charalampopoulos, S.P. Pissis, **R**: Dynamic and Internal LCF. Algorithmica 2020

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```
c b b a a b c a b a a b c b a

d a a b a c a b c b b a
```

- $\log^{O(1)} n$-time "after-edit" queries with $\tilde{O}(n)$-sized data structure [1]

  $\leftarrow$ HIA queries
- fully dynamic $\tilde{O}(n^{2/3})$-time updates [2] $\leftarrow$ dynamic strings, MaxPairLCP
- fully dynamic $\log^{O(1)} n$ amortized time substitutions [3]

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- input strings keep changing: insertions, deletions, substitutions allowed
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c b b a a b c a b a a b c b a

d a a b a c a b c b b a

- $\log^{O(1)} n$-time "after-edit" queries with $\tilde{O}(n)$-sized data structure [1]

  $\leftarrow$ HIA queries
- fully dynamic $\tilde{O}(n^{2/3})$-time updates [2] $\leftarrow$ dynamic strings, MaxPairLCP
- fully dynamic $\log^{O(1)} n$ amortized time substitutions [3] $\leftarrow$ locally cons. parsing

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Internal LCF

- queries about LCF of given factors of two strings

c b **b a a b c** a b c a b c b a

d a a **b a a b c** b b a

Internal LCF

- queries about LCF of given factors of two strings

c b b a a b c a b c a b c b a

d a a b a a b c b b a

Internal LCF

- queries about LCF of given factors of two strings

c b b a a b c a b c a b c b a

d a a b a a b c b b a

Internal LCF

- queries about LCF of given factors of two strings

c b b a a b c a b c a b c b a

d a a b a a b c b b a

factor of S	factor of T	space	query	comment
any	any	$\tilde{O}(n^2/t^2)$	$\tilde{O}(t)$	
any	any	$\tilde{\Omega}(n^2/t^2)$	t	clb, Set Disjointness

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c b b a a b c a b c a b c b a

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factor of S	factor of T	space	query	comment
any	any	$\tilde{O}(n^2/t^2)$	$\tilde{O}(t)$	
any	any	$\tilde{\Omega}(n^2/t^2)$	t	clb, Set Disjointness
prefix/suffix	prefix/suffix	$O(n \log n)$	$O(\log n)$	$O(n \log^2 n)$ constr.
any	whole T	$O(n)$	$O(\log n)$	$O(n)$ constr.

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c b b a a b c a b c a b c b a

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factor of S	factor of T	space	query	comment
any	any	$\tilde{O}(n^2/t^2)$	$\tilde{O}(t)$	
any	any	$\tilde{\Omega}(n^2/t^2)$	t	clb, Set Disjointness
prefix/suffix	prefix/suffix	$O(n \log n)$	$O(\log n)$	$O(n \log^2 n)$ constr.
any	whole T	$O(n)$	$O(\log n)$	$O(n)$ constr.

OP: Better data structures for special cases?

[1] A. Amir, P. Charalampopoulos, S.P. Pissis, **R**: Dynamic and Internal LCF. Algorithmica 2020

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Other Variants of LCF

- Longest common Abelian factor (LCAF) [1-5]

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[6] M. Crochemore, C.S. Iliopoulos, T. Kociumaka, M. Kubica, A. Langiu, S.P. Pissis, **R**,
W. Rytter, T. Waleń: Order-preserving indexing. Theor. Comput. Sci., 2016

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- Longest common circular factor (LCCF) [7]

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[7] M. Alzamel, M. Crochemore, C.S. Iliopoulos, T. Kociumaka, **R**, W. Rytter, J. Straszyński,
T. Waleń, W. Zuba: Quasi-Linear-Time Algorithm for LCCF. CPM 2019

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